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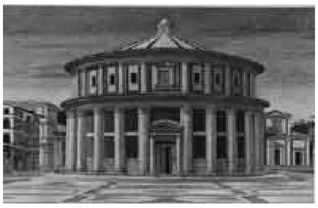

Notes on Hintikka's Analytic-Synthetic Distinction

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Abstract

Hintikka's most original contribution to the debate on the analytic-synthetic distinction is probably his insight that synthetic arguments, unlike analytic ones, introduce new individuals into the discussion. This intuition is not only the kernel of his theory, but it is also both an interpretation of Kant's conception of the mathematical method and, at the same time, an attack through formal means against the logical empiricists' tenet that logic is analytic and tautological. This paper provides: 1. a reconstruction of Hintikka's overall picture that takes into account both the philosophical and the formal part of his work; 2. an analysis of the different components of Hintikka's conception and of their relations; 3. a series of objections against Hintikka's theory that points towards an alternative development of the same basic idea.

Keywords: Hintikka, Kant, analytic-synthetic distinction, distributive normal forms, Depth Bounded First-Order Logics

1. Introduction

In his paper *An Analysis of Analyticity*[1], Hintikka discusses four main senses in which the analytic-synthetic distinction has been understood against the backdrop of the Kantian use of the terms:

i. analytic truths as conceptual truths;

[1] Hintikka (1966), Hintikka (1973, 123-49). References will always be made to the latter version of the text.

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ii. analytic arguments as arguments satisfying some form of the subformula property;
iii. analytic arguments as arguments that do not introduce new individuals into the discussion;
iv. analytic truths as tautological truths.

His work has some indisputable merits. It debates traditional senses of analyticity, such as i and iv, and points out definitions that deserve more attention from the philosophical side, such as ii. Nevertheless, Hintikka's most original contribution to the subject is contained in the third conception of analyticity.

Sense iii is not only the innovative kernel of Hintikka's conception of the analytic-synthetic distinction. It is also both an interpretation and a vindication of Kant's theory of the mathematical method[2] and, at the same time, an attack, conducted through modern formal means, against the logical positivistic traditional tenet that logic is analytic and tautological[3]. Although these three issues are strongly interconnected in Hintikka's thought, each piece of the secondary literature is surprisingly circumscribed to one or the other of them. Papers have been written on Hintikka's notion of syntheticity in terms of the introduction of new individuals soon after the appearance of *Logic, Language-Games and Information*[4]; remarkable attention has been paid on Hintikka's reading of the Kantian materials, which has raised a great deal of debate especially as far as the notion of intuition is concerned[5]; few and precise contributions have been produced on the theory of distributive normal forms for first-order logic[6]. But a comprehensive interpretation of Hintikka's overall project on the analytic-synthetic distinction, whose presentation is fragmented into a considerable number of papers[7], is still lacking.

The main contributions of this paper are: 1. a reconstruction of Hintikka's overall theory that takes into account both the philosophical and the formal part of his work; 2. an analysis of the different components of

[2] See, e.g., Hintikka (1973, 174-98, 199-221), Hintikka (1968) and Hintikka (1967).
[3] See, e.g., Hintikka (1973, 222-41, 242-86), Hintikka (1970) and Hintikka (1965a).
[4] See, e.g., Van Benthem (1974).
[5] See, e.g., De Jong (1997), Russell (1990) and Parsons (1969).
[6] See, e.g., Rantala (1987), Rantala and Tselishchev (1987) and Sequoiah-Grayson (2008). The scarse success of Hintikka's theory of distributive normal forms for first-order logic is probably due to its complexity (see Section 6). It might be the case that this in turn has had an important impact of the fortune of Hintikka's overall theory of analyticity.
[7] Some of them are collected in Hintikka (1973). For the others, see Kolak and Symons (2004).

Hintikka's conception and of their relations; 3. a series of objections against Hintikka's developments of his sense iii.

This paper is organized as follows. Section 2 analyses Hintikka's variations of sense iii. Sections 3 and 4 examine in a critical way Hintikka's interpretation of Kant's notion of intuitions as singular representations and of Kant's conception of analysis respectively. Section 5 focuses on the problems concerning the relation between Hintikka's sense iii and his reading of the Kantian materials. Section 6 offers a simple presentation of Hintikka's theory of distributive normal forms for first-order logic, which grounds the thesis that some logical inferences are synthetic and informative. This thesis, together with the relationship with Kant's theory and the logical empiricists' standpoint, is discussed in Section 7. Section 8 points out ten difficulties that affects Hintikka's overall picture: some of them concern the various components of his theory taken separately, other the relations between them. Section 9 concludes by suggesting an alternative development of Hintikka's sense iii that overcomes the difficulties highlighted in this paper.

2. Hintikka's sense iii

Hintikka's sense iii assumes that what the premises of an argument gives us to be analysed are *individuals*, rather than *concepts*. In so doing, it allows the application of the analytic-synthetic distinction to that modern and symbolic first-order logic that results from Frege's revolution based on the rejection of the traditional subject-concept schema in favour of the argument-function analysis of judgments[8]. Hintikka elaborates this conception of analyticity providing further specifications of his sense iii, which he understands as variations of the same basic idea:

> iii. A (valid) argument step is analytic if it does not introduce new individuals into the discussion.
> iii.a. An analytic argument cannot lead from the existence of an individual to the existence of a different individual […].
> iii.b. A (valid) argument step is analytic if it does not increase the number of individuals one is considering in relation to each other.
> iii.c. A (valid) argument step is analytic if the degree of the conclusion is no greater than the degree of at least one of the premises.
> iii.d. An argument is analytic if all its steps are analytic in sense iii.c.

[8] Frege (1879).

> iii.e. A (valid) proof of the sentence F_1 from F_0 is analytic if none of the sentences occurring as intermediate stages of this proof has a higher degree than F_0 and F_1[9].

Sense iii.a does not seem to distance itself significantly from iii. It simply rests on the plausible assumption that introducing an individual into an argument is tantamount to affirming its existence. On the contrary, although Hintikka does not stress this difference, sense iii.b. specifies, against iii, that argument steps in which either unrelated individuals are introduced, or the number of new related individuals that are introduced is the same as the number of old related individuals that are removed, still count as analytic.

But it is only with sense iii.c that Hintikka gives a precise definition of what it means for certain individuals to be in relation to each other in a given sentence. His starting point is the observation that not only free singular terms, but also quantifiers invite us to consider different individuals[10]. This is confirmed by the fact that for many purposes quantifiers may be omitted, and the variables bound to them replaced by free singular terms. However, it is not the case that every single quantifier recalls a distinct individual:

> […] parallel quantifiers (i.e., quantifiers whose scopes do not overlap) cannot be said to add to the number of individuals we have to consider in their relation to one another. They may introduce new cases to be considered, but they do not complicate the complexes of individuals we have to take into account[11].

Accordingly, Hintikka defines[12] the maximal number of individuals that are in relation to each other in a certain sentence F, called the *degree of* F, as the sum of two numbers:

1. the number of the free singular terms of F; and
2. the maximum length of nested sequences of quantifiers in F, called the *depth of* F, $d(F)$, and recursively defined as follows:
    - $d(F) = 0$ for F atomic;
    - $d(\neg F) = d(F)$;
    - $d(F_1 \wedge F_2) = d(F_1 \vee F_2) = d(F_1 \rightarrow F_2) = \max\big(d(F_1), d(F_2)\big)$;
    - $d(\forall x F) = d(\exists x F) = d(F) + 1$.

[9] Hintikka (1973, 148-9).
[10] Hintikka (1973, 138 and ff.).
[11] Hintikka (1973, 140).
[12] Hintikka (1973, 141-2) with minor modifications.

The depth of a sentence F, which is the number of different layers of quantifiers in F at its deepest, is just the number of bound individual symbols that are needed to understand F, provided that quantifiers with overlapping scopes always have different variables bound to them. The definition makes clear that the depth of a sentence is not the number of individuals of which the sentence speaks. As Hintikka specifies[13], in the sentence $All\ men\ admire\ Buddha$, $\forall x(Mx \rightarrow Axb)$, the number of individuals considered in their relation to each other is two (Buddha, b, and each man at a time, just take an arbitrary individual, say a, to instantiate the variable x), while reference is somehow made to all individuals in the domain.

Sense iii.d extends sense iii.c from argument steps to longer arguments. Sense iii.e departs quite decisively from iii.d. For Hintikka, the former is «the more interesting and important of the two»[14]. Unlike iii.d, sense iii.e does not regard the direction of the argument and takes into account also the individuals occurring in the conclusion. As a result, any inference in which the degree of the conclusion is higher than the degree of the premise is synthetic according to sense iii.d, but might turn out analytic according to sense iii.e.

3. Kantian intuitions as singular representations

As anticipated above, Hintikka's sense iii claims to be a vindication of Kant's conception of analyticity. It is clear that in order to vindicate a certain theory, it is necessary, first of all, to understand and interpret it properly. But is it really the case that Hintikka's interpretation of Kant's theory of analyticity is accurate and faithful to the original texts? Hintikka's reading of the Kantian materials is based on two main elements. First, the nature of Kant's intuitions: Hintikka holds that intuitions are defined as singular representations and stand for individual objects. Second, Kant's conception of syntheticity: Hintikka claims that Kant is an «heir to the constructional sense of analysis»[15] and that the notion of syntheticity is founded on the necessary employment of constructions.

This section considers the first point. Hintikka's well-known interpretation of Kant's notion of mathematical intuitions can be reconstructed as consisting in the following moments.

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[13] Hintikka (1965a, 187).
[14] Hintikka (1973, 193).
[15] Hintikka (1973, 205).

1. There are two stages in the development of Kant's conception of mathematical intuition. The former is reflected also in the *Introduction* and in the *Doctrine of Method* of the first *Critique*; the latter is represented by the *Transcendental Aesthetic*. The former stage is logically prior to the latter. This thesis is defended on the basis of textual elements[16].
2. In the former stage, mathematical intuitions are simply defined as representations of individuals. In Hintikka's words: «For Kant, an intuition is simply anything which represents or stands for an individual object as distinguished from general concept»[17] and «Everything […] which in the human mind represents an individual is an intuition»[18].
3. In the latter stage, mathematical intuitions are connected to sensibility in order to explain why mathematical results obtained through the employment of intuitions can be applied to all kinds of experience.
4. The former stage can be detached from the latter. Kant's assumption that sense-perception is the process by means of which we become aware of the existence of individuals is considered by Hintikka as «Kant's basic fallacy in the first *Critique*»[19]. In assuming the connection between sensibility and individuality, Kant would have surrendered to the classical tradition at the expense of his own principles[20].

Hintikka's interpretation of Kant's mathematical intuitions has been at the core of an enduring debate, a famous chapter of which is represented by Charles Parsons' criticism. Against Hintikka's thesis that the essential feature of intuition is individuality, Parsons maintains that intuitions are defined on the basis of the immediacy criterion[21]. Both of the two features invoked find textual confirmation in the *Critique*, where Kant, defining intuitions in contrast to concepts, affirms «The former [i.e., intuition] is immediately related to the object and is singular; the latter [i.e., concept] is mediate, by means of a mark, which can be common to several things»[22].

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[16] See Hintikka (1967, 356 and ff.).
[17] Hintikka (1974, 130).
[18] Hintikka (1967, 354-5).
[19] Hintikka (1984, 102).
[20] Hintikka (1987, 29).
[21] See, e.g., Parson (1969), Parson (1980) and Parson (1983).
[22] Kant (1998, A320/B377).

On the one side, Hintikka's position about the relationship between the two criteria remains essentially anchored to the notion of individuality. Initially, Hintikka does not distinguish the two features and holds that «another way of saying that *Anschauungen* have an immediate relation to their objects is to say that they are particular ideas»[23]; then, he maintains that immediacy is just a «corollary»[24] of the individuality criterion, because immediacy is the proper mode of reference of singular objects. On the other side, Parsons argues that the reason for which Kant added the feature of immediacy next to that of individuality in his definition of intuition is that the former is significantly different from the latter. The scholar describes immediacy in perceptual terms, as a phenomenological presence to the mind, and accuses Hintikka's reading of underestimating the spatio-temporal content of intuition and the distinctive role it plays in mathematics.

Hintikka and Parsons' positions, which have come to be known respectively as the *logical* and *phenomenological* interpretations of Kant's intuitions[25], have been recently accused of being too radical. For example, Michael Friedman expresses the belief that «this dichotomy is artificial» and that «a truly adequate interpretation of Kant's philosophy of mathematics […] must make room for elements from both the […] approaches»[26]. The strongest reason for charging Hintikka of extremism is probably the way in which he treats the Kantian texts, because, as Parsons rightly notices, «[m]any of the passages Hintikka cites also mention the immediacy criterion, and it is not clear why Hintikka thinks it nonessential»[27].

Consider a single but telling example. In supporting step 1 of his reconstruction, Hintikka maintains that in the *Transcendental Aesthetic* Kant's reasoning proceeds from the assumption that intuitions are individual representations. Nevertheless, Kant opens that chapter of his work with a clear reference to the immediacy criterion, which is completely overlooked by Hintikka: «In whatever way and through whatever means a cognition may relate to objects, that through which it relates immediately to them, and that which all thought as a means is directed as an end, is intuition»[28]. Although in a later article Hintikka complains that the objections against his interpretation paid no attention to the doctrinal context, but rather focused

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[23] Hintikka (1969, 42).
[24] Hintikka (1982, 202).
[25] See Friedman (2000).
[26] Friedman (2010, 586).
[27] Parsons (1969, 570).
[28] Kant (1998, A19/B33).

on his reading of particular passages[29], textual criticisms of this kind cannot be dismissed as pedantic remarks. On the contrary, these passages play a fundamental role in a proper understanding of the Kantian conceptions.

4. «An heir to the constructional sense of analysis»

According to Hintikka, the model Kant employs in developing his theory of mathematics is provided by Euclid's system of elementary geometry. Following Proclus[30], the proof of a theorem and the solution of a problem in Euclid's *Elements* usually consisted of six main parts: 1. enunciation or *protasis* of the general proposition; 2. setting-out or *ecthesis* that is the exhibition of a certain figure, to which the general proposition is applied, whose particular determinations are completely indifferent to the proof of the proposition; 3. definition or specification (*diorismos*), in which the figure set out in the *ecthesis* is determined more precisely; 4. machinery or *kataskeue*, which consists in completing the construction drawn in the setting-out; 5. proof proper or *apodeixis*; 6. conclusion or *superasma*, namely the generalization of the conclusion from the particular figure to the general case.

Not only does Hintikka recognize that most of these steps turn up also in Kant's conception of the geometrical procedures; he also maintains that Kant's overall picture of the mathematical method is influenced by two specific moments: the *ecthesis* and the machinery. The centrality of the setting-out is demonstrated by Kant's idea that mathematics cannot deal with general concepts, but needs to reason *in concreto* and nonetheless reaches results that hold in general[31]. The relevance of the machinery is shown by Kant's claim that the mathematical method proves to be better than philosophical procedures in certain tasks (such as determining the relation between the sum of the angles of a triangle and right angle) in virtue of the fact that geometers, unlike philosophers, can draw actual figures and carry out proofs in terms of such figures[32].

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[29] Hintikka (1982, 202-3).

[30] The relation between Proclus' text and the ancient Greek geometrical practice is not clear. Netz (1999) argues that the Proclean scheme was invented for the purpose of a commentary on Euclid, since its terminology cannot be found before Proclus himself. Acerbi (2019) shows that Proclus' text includes all the ingredients that contribute to the traditional explanation of the generality of Euclidean proofs, which, nevertheless, falsifies crucial syntactical elements and employs an incorrect conception of the denotative letters.

[31] Kant (1998, A715-6/B743-4).

[32] Kant (1998, A716-7/B74).

What is exhibited by the two processes of *ecthesis* and machinery are *constructions*, which, for Hintikka, are the essence of the geometrical method in Kant's theory. But that's not all: Hintikka holds that for Kant constructions are the characteristic features not only of geometry, but also of mathematics as a whole[33] and, more crucially, of the notion of syntheticity in general. While the application of the Euclidean paradigm to arithmetic and algebra can be easily justified by appealing to Descartes' new geometry[34], Kant's supposed leap from the mathematical method to the analytic-synthetic distinction *tout court* is the most delicate conceptual step in Hintikka's interpretation[35]. In order to reach the conclusion that construction is the fundamental notion at the basis of Kant's analytic-synthetic distinction, Hintikka has to show that Kant held the following principles:

P1. The use of synthetic methods in their proof makes judgments synthetic;
P2. The use of constructions makes methods synthetic.

Although he is neither explicit in assuming it, nor careful in justifying his ascription to Kant, P1 is a widespread and essential element in Hintikka's interpretation[36]. It amounts to suppose that the Kantian distinction between analytic and synthetic judgments is founded on the distinction between analytic and synthetic methods. Nevertheless, as Willem De Jong rightly emphasizes, albeit P1 is «fairly plausible» in the domain of mathematics, «in the light of Kant's entire philosophical edifice a serious difficulty immediately presents itself: namely, mutatis mutandis a number of things seem to fit in much less well for metaphysics or philosophy»[37].

But what about P2? Hintikka explicitly argues for this thesis in at least two occasions. First, in his paper *An Analysis of Analyticity*[38], Hintikka claims that although there is a certain similarity between Kant's notion of analyticity and analyticity as conceptual truth (sense i), «this similarity is largely an illusion». On the contrary, according to Hintikka, «sense iii approximates rather closely Kant's notion of analyticity». Second, in his paper *Kant and the Tradition of Analysis*[39], Hintikka maintains that Kant is

[33] Kant (1997, 20).
[34] Hintikka (1982, 206).
[35] See also De Jong (1997).
[36] See, e.g., Brittan (2015, 55).
[37] De Jong (1997, 146).
[38] Hintikka (1973, 123-49).
[39] Hintikka (1973, 199-221).

«an heir to the constructional sense of analysis»[40], which amounts again to sense iii, and not of the directional one. According to the latter, analysis is understood in the regressive sense as the route in the direction of the principles or axioms: it assumes the desired end and argues backwards; synthesis follows instead a progressive path from principles towards what is grounded on them[41]. In both papers, Hintikka argues for P2 on the basis of textual evidence. But, again, his reading of the passages proposed is not overwhelming.

Consider, for example, the text taken from a footnote of the *Prolegomena*[42], where Kant is pointing out the risks that arise when traditional terms are employed for describing new concepts: here, he distinguishes between the meanings that the words *analytic* and *synthetic* assume if applied to judgments rather than methods and proposes, in order to avoid confusions, to refer to the latter with the adjectives *regressive* and *progressive*. From the fact that the two methods are described through directional terms, Hintikka concludes that, in applying the analytic-synthetic distinction to judgments, Kant is referring to the constructional tradition of the terms. Now, Hintikka's conclusion not only goes against P1, but it also assumes that there are only two traditions of analyticity to which Kant could appeal. Moreover, as De Jong[43] underlines, it does not take Kant at his words when he says that his distinction between analytic and synthetic judgments founds a new meaning of the terms.

Another text that Hintikka calls into question is a particularly debated passage of the *Critique*[44] where Kant maintains that «the inferences of the mathematicians all proceed in accordance with the principle of contradiction» and, in so doing, seems to be at odds with his own claim that the principle of contradiction is «the supreme principle of analytical judgments»[45]. Hintikka holds that B14 can be clarified along the

[40] Hintikka (1973, 205).
[41] In his work with Unto Remes, Hintikka regards the directional sense of analysis and synthesis as «a pale reflection of the richness of the ideas involved in the original Greek concepts» and claims that it «has nothing to do with the heuristic method of analysis» (Hintikka and Remes (1974, 11 and 19)). The preoccupation with the direction of analysis has caused the serious side effect that subtler ingredients of the Greek method of analysis and synthesis have been overlooked for centuries, at least until the end of the Middle Ages. This is the case also of the role of constructions and the importance of the *machinery* needed to carry out the demonstration. See also Hintikka and Remes (1976).
[42] Kant (1997, 28).
[43] De Jong (1997, 159).
[44] Kant (1998, B14).
[45] Kant (1998, A151-2/B190-1).

constructional interpretation, if it is understood as referring only to a single part in the proof of a proposition, namely, the *apodeixis*: the proof proper is in fact analytic, because the synthetical element of a geometrical proposition rests only with the auxiliary constructions. Hintikka's reading seems to be consistent, but nevertheless it is not decisive. For example, according to the *evidentialist* interpretation[46], Kant is appealing here to the syntheticity of the axioms of mathematics as a clarification for the syntheticity of all mathematical theorems: this would explain Kant's claim that «a synthetic proposition can of course be comprehended in accordance with the principle of contradiction, but only insofar as another synthetic proposition is presupposed from which it can be deduced, never in itself». Another interesting perspective is that in this passage Kant is underlining, against the Leibnizians, that the principle of non-contradiction can be used only as a negative criterion of truth.

Thus, neither P1, nor P2 have been properly justified by textual evidence. This leads to conclude that the connection Hintikka wanted to establish between Kant's theory of the mathematical procedures and his distinction between analytic and synthetic judgments is not free of difficulties.

5. *Ecthesis* and existential instantiation

Hintikka claims that his sense iii is a vindication of Kant's analytic-synthetic distinction. Sections 3 and 4 have shown that sense iii is a vindication not so much of Kant's analytic-synthetic distinction, but, if anything, of Hintikka's reading of this distinction. But is it really the case that sense iii «approximates rather closely»[47] the Hintikkian interpretation of Kant's theory? Hintikka's claim can be reconstructed by way of the analogies displayed in Table 5.1.

Levels of Hintikka's theory	**Ways to introduce objects**	**Objects introduced**
Interpretational premises	*Kant is «an heir to the constructional sense of analysis»*	*Kantian intuitions are defined by the singularity criterion*
1. Hintikka's reconstruction of Kant's geometrical method	*Ecthesis* = exhibition of a certain figure	Geometrical figure

[46] Brittan (2006).
[47] Hintikka (1973, 137).

2. Hintikka's reading of Kant's synthetic method	Construction = *a priori* exhibition of an intuition	Intuition as a singular representation
3. Hintikka's interpretation of modern first-order logic	Rule of existential instantiation = introduction of a new individual	Individual in the domain

Table 5.1: Reconstruction of Hintikka's claim that sense iii «approximates rather closely» the Hintikkian reading of Kant's theory.

According to Hintikka's reconstruction of Kant's thought, the syntheticity of geometrical proofs rests on the exhibition of certain figures through the step called *ecthesis*. As it has been shown above, Hintikka extends this insight by holding that it is the use of constructions that makes whatever method synthetic. This generalization is based on the idea that geometrical constructions are analogous to another wider and abstract notion of construction that Kant employs throughout his work. This second concept of construction is founded on the notion of intuition and is referred to by Kant as follows: «to construct a concept means to exhibit a priori the intuition corresponding to it»[48]. The analogy between the two notions of construction is quite evident: *ecthesis* introduces into the argument geometrical figures, which are singular and individual objects, in order to represent the general proposition; similarly, constructions in general introduce into the argument intuitions, which are singular objects as well, in order to represent a general concept[49].

But Hintikka pushes this similarity further. While synthetic methods make use of constructions that exhibit *a priori* intuitions understood as singular representations, synthetic arguments of modern first-order logic introduce new individuals into the argument through the applications of the rule of existential instantiation. By this rule, a free individual symbol is introduced to replace the occurrences of a certain bound variable: this rule allows to infer from an existentially quantified sentence $\exists xF$ a sentence instantiating it, e.g. $F(x/a)$, where a is a free individual symbol and $F(x/a)$ the result of replacing x by a in F. The individual introduced in the proof through an application of this rule is new because of the requirement that the instantiating symbol a must be different from all the free individual symbols occurring earlier in the proof. Thus, according to Hintikka, the rule

[48] Kant (1998, A713/B741).

[49] See Acerbi (2011) and Acerbi (2019) on the generality of the entities involved in a geometric proof of the kind found in ancient Greek treatises.

of existential instantiation is the paradigmatic synthetic method in first-order logic[50] and *ecthesis* becomes «identical»[51] with it.

It is worth noting that the analogy between geometrical figures, intuitions and individuals in the logical domain is possible by virtue of Hintikka's definition of the Kantian intuitions through the sole means of the singularity criterion; while the similarity between *ecthesis*, use of constructions and application of the rule of the existential instantiation is possible by virtue of Hintikka's ascription of Kant's notion of analysis to the constructional tradition. This means that the similarity between sense iii and Kant's theory is based on the most difficult passages of Hintikka's interpretation of the Kantian texts.

But this is not the only problem that affects Hintikka's identification of sense iii with Kant's analytic-synthetic distinction. For example, some inferences that are synthetic according to Kant turn out to be analytic following Hintikka's favorite version of sense iii, namely sense iii.e. One of these inferences is Kant's famous judgement $7+5=12$, which is discussed by Hintikka himself[52]. Cases like these might be easily dismissed by accepting, as Allen Hazen does, that «logical theory has advanced too much since Kant's days for his views to have any precise application to it»[53]. What is perhaps harder to reconcile are the difficulties regarding Hintikka's analogy between Kant's *ecthesis* and the use of the existential instantiation in modern first-order logic.

To begin with, the process that takes place during the *ecthesis* should be better represented by the rule of universal instantiation, rather than existential instantiation. Geometrical theorems of Euclid's *Elements* usually ask to prove that a certain property is satisfied by all the objects belonging to a certain class. Through the *ecthesis*, one single figure is chosen among this class. Similarly, the rule of universal instantiation allows to infer from a universally quantified sentence $\forall x F$a sentence instantiating it, e.g. $F(x/a)$, where a is any free individual term. Now, the requirement that the figure chosen through the setting-out must not have particular determinations affecting the proof of the proposition is something that is not necessary *per se*, but only in order to extend the results obtained for a particular figure to all the figures of a certain kind. In logical terms, the requirement that the term a above is arbitrary is needed in order to apply the rule of universal generalization. The reason why Hintikka speaks of existential instantiation

[50] See, e.g., Hintikka (1965a, 199) and Hintikka (1973, 210).
[51] Hintikka (1967, 368-9).
[52] Hintikka (1973, 194-5).
[53] Hazen (1999, 97).

instead of the more natural universal instantiation is probably that while the individual introduced by the former must always be fresh, the individual introduced by the latter can also already occur above. This point would have upset Hintikka's analogy between Kant's geometrical method and first-order logic.

Moreover, Kant would probably have conceived every single application of both the universal and the existential instantiation as synthetic steps. For him, arguments are made synthetic by the very introduction of intuitions[54]. But this cannot be Hintikka's perspective. Consider, for example, senses iii.b and iii.c. These versions suggest that applications of the rule of the existential instantiation are not synthetic argument steps. In this rule, a new free singular term is introduced, but, at the same time, an existential quantifier is removed. As a result, an argument step of this kind is analytic, because it does not increase the number of individuals one is considering in relation to each other and the degree of the conclusion is no greater than the degree of the premise. Another proof that Hintikka has not accepted Kant's point is that otherwise all of first-order logical inferences, in which quantificational reasoning plays an essential role, would have turned out to be synthetic. Nevertheless, as it will be shown in the next sections, Hintikka's rejection of the logical positivistic tenet that logic is analytic is confined to a limited class of first-order logical inferences.

6. Distributive normal forms for first-order logic

In the manifesto of the Vienna Circle, written in 1929, Rudolph Carnap, Hans Hahn and Otto Neurath claimed that logic is analytic: «The scientific world-conception knows only empirical statements about things of all kinds, and analytic statements of logic and mathematics»[55]. Not only does the logical empiricists' movement hold that logic is analytic. It also claims that logic is tautological, namely trivial and devoid of any informational content. This is the side effect of the paradox of analysis[56], which states that analysis cannot be sound and informative at the same time: for if it is sound, the analysed and the analysandum are equivalent and analysis cannot be augmentative; and if it is informative, then the analysed and the analysandum are not equivalent and the analysis is incorrect.

[54] Hintikka (1973, 194).
[55] Carnap, Hahn and Neurath (1973, 311).
[56] Langford (1992, 323).

Starting from the Church-Turing result that classical first-order logic is undecidable[57], Hintikka attacks both the thesis that logic is analytic and the thesis that logic is tautological. On the one hand, Hintikka provides a technical definition of his sense iii, according to which a class of polyadic first-order inferences are synthetic. On the other hand, he devises a new notion of information, which he calls *surface information*, that might be increased by logical deduction. In order to support both of the claims, he makes use of the theory of distributive normal forms, which he elaborates as an alternative to Carnap's notion of states description[58].

The theory of distributive normal forms for first-order logic, the elementary notion of which are outlined below[59], is an extension of the corresponding theory for propositional logic and monadic first-order logic. It provides a description of possible worlds, where the complexity of the configurations of individuals that can be considered is limited. Let F be a first-order formula characterized by the following parameters:

P1. the set of all the predicates occurring in it;
P2. $\{a_1, \dots, a_k\}$, which is the set of all the free individual symbols occurring in it;
P3. d, which is the maximal length of sequences of nested quantifiers occurring in it (its depth).

The distributive normal form of F with the same parameters is a disjunction of conjunctions called *constituents*, C_i^d:

$$F^d \equiv DNF^d(F) = \bigvee_{i=1}^{j} C_i^d(a_1, \dots, a_k). \tag{6.1}$$

Each constituent $C_i^d(a_1, \dots, a_k)$, characterized by certain parameters P1, P2 = $\{a_1, \dots, a_k\}$ and P3 = d, represents a possible world that can be described by the sole means of these parameters. The disjuncts that occur in the distributive normal form of the formula F represent all and only the descriptions of the possible worlds that are not excluded by F. Before giving

[57] Church (1936) and Turing (1937).
[58] Carnap (1956).
[59] This presentation is based on Hintikka (1953), Hintikka (1965b) and, especially, Hintikka (1973, 242-86).

a definition of the constituents, it is first necessary to explicate some notations and to define the crucial notion of *attributive constituent*. Given a set of predicates P1, let $A_i(a_1, \dots, a_k)$ be all the atomic formulae that can be formed by the members of P1 and from the individual symbols $\{a_1, \dots, a_k\}$ and let $B_i(a_1, \dots, a_k)$ be all the atomic formulae so defined that contain at least one occurrence of a_k. It follows[60] that:

$$\bigwedge_{i=1}^{r} A_i(a_1, \dots, a_k) = \bigwedge_{i=1}^{s} A_i(a_1, \dots, a_{k-1}) \wedge \bigwedge_{i=1}^{t} B_i(a_1, \dots, a_k). \tag{6.2}$$

An attributive constituent, written $Ct^d(a_1, \dots, a_k)$, with parameters P1, P2 = $\{a_1, \dots a_k\}$ and P3 = d can be recursively defined in terms of attributive constituents of depth $d - 1$ as follows[61]:

$$Ct_r^d(a_1, \dots, a_k) = \bigwedge_{i=1}^{s} B_i(a_1, \dots, a_k) \wedge \bigwedge_{i=1}^{t} \exists x\, Ct_i^{d-1}(a_1, \dots, a_k, x). \tag{6.3}$$

Intuitively speaking, if constituents represent possible worlds, attributive constituents describe possible kinds of individuals. The attributive constituent shown in the expression 6.3 represents the complex attribute of the individual referred to by a_k and describes this individual by the sole means of P1, the "reference-point" individuals $a_1, \dots, a_k$ and at most d layer of quantifiers. It is necessary to examine the components of the equation 6.3 one by one. The first conjunct $\bigwedge_{i=1} s\ B_i(a_1, \dots, a_k)$ specifies the way in which a_k is in relation with the individuals $a_1, \dots, a_{k-1}$: it is a conjunction of all the atomic formulae that can be formed from P1 and P2 that contain a_k. Then, for every i, $Ct_i^{d-1}(a_1, \dots, a_k, x)$ provides a list of all the kinds of individuals that can be specified through the parameters P1, the individuals $\{a_1, \dots, a_k\}$ and $d - 1$ layers of quantifiers. For each such kind of individual, the expression $\bigwedge_{i=1} t\ \exists x$ specifies whether individuals of that particular kind exist or not.

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[60] For each given r and for suitably chosen (with respect to r) s and t.

[61] Here it is assumed that indices are used to distinguish different attributive constituents with the same parameters from one another. The index r is a function of s and t. This dependence will be assumed also in the expressions that follow.

The notion of a constituent with parameters P1, P2 = $\{a_1, \dots, a_k\}$ and P3 = d is simply defined on the basis of attributive constituents with the same parameters as follows[62]:

$$C^d(a_1, \dots, a_k) = \bigwedge_{i=1} A_i(a_1, \dots, a_{k-1}) \wedge Ct^d(a_1, \dots, a_k). \tag{6.4}$$

As a simple example to clarify the definitions above, consider the distributive normal form of the sentence *All men laugh*, namely $DNF^1(\forall x(Mx \rightarrow Lx))$. Here, P1 = $\{L, M\}$, P2 = $\emptyset$ and P3 = 1[63]. The different kinds of individuals specifiable by means of the two predicate expressions M and L are the attributive constituents listed below:

$$\begin{aligned} Ct_1^0(x) &= Mx \wedge Lx \\ Ct_2^0(x) &= Mx \wedge \neg Lx \\ Ct_3^0(x) &= \neg Mx \wedge Lx \\ Ct_4^0(x) &= \neg Mx \wedge \neg Lx. \end{aligned} \tag{6.5}$$

Now, each kind of possible world that can be described by means of the predicates in P1 is specified by indicating, for each kind of individual, whether there are such individuals in the world in question. So, for example, the constituent of depth 1:

$$\begin{aligned} C_4^1 = {} & \exists x(Mx \wedge Lx) \wedge \exists x(Mx \wedge \neg Lx) \wedge \neg\exists x(\neg Mx \wedge Lx) \\ & \wedge \neg\exists x(\neg Mx \wedge \neg Lx) \end{aligned} \tag{6.6}$$

describes a world in which there are individuals that are men and laugh; there are individuals that are men and do not laugh; there are no individuals that are not men and laugh; and there are no individuals that are not men and do not laugh. It turns out that the distributive normal form at depth 1 of the sentence *All men laugh* is the disjunction of all the constituents in which

[62] Here the indices of C^d, $\wedge$ and Ct^d have not been specified, but of course the first depends on the other two.

[63] Notice that in this case, $A_1(a_1, \dots, a_k)$ and $B_i(a_1, \dots, a_k)$ are clearly empty and, as a result, for every j, $C_j^d = Ct_j^d$.

there are no individuals that are men and do not laugh, namely constituents of the form:

$$C_j^1 = (\pm)\exists x(Mx \wedge Lx) \wedge \neg\exists x(Mx \wedge \neg Lx) \wedge (\pm)\exists x(\neg Mx \wedge Lx) \wedge (\pm)\exists x(\neg Mx \wedge \neg Lx), \tag{6.7}$$

where the $(\pm)$ notation is used in the obvious way. This concludes the example.

Hintikka proves that every first-order formula F with certain parameters can be converted into its distributive normal form with the same parameters or with certain fixed larger ones. As a special case, every constituent with depth d and some parameters P1 and P2 can be converted into a disjunction of constituents, called *subordinate*, with the same parameters P1 and P2 but greater depth $d + e$ for some natural number e.

This observation is crucial for the distinction between inconsistent constituents that are trivially inconsistent and inconsistent constituents that are not trivially inconsistent. While the former are blatantly self-contradictory and satisfy some of the inconsistency conditions put forward by Hintikka, the inconsistency of the latter can be detected only by increasing their depth. In other words, it is shown that for every inconsistent constituent C^d there is some natural number e such that all the subordinate constituents of depth $d + e$ are trivially inconsistent. However, due to the Church-Turing result that first-order logic is undecidable, it is not known which depth must be reached for acknowledging that a certain constituent is inconsistent.

Thus, in order to prove G from F, it is necessary to combine the parameters P1 and P2 of F and G, take the maximum, say d, of their depths, and convert F and G into their distributive normal forms F^d and G^d with the parameters just obtained. Then, F^d and G^d must be expanded by splitting their constituents into disjunctions of deeper and deeper constituents and at each step all the trivially inconsistent constituents must be omitted. Then, if G follows from F, there will be an e such that all the non-trivially inconsistent members of F^{d+e} obtained by this procedure are among the non-trivially inconsistent members of G^{d+e} obtained through the same procedure. A proof of G from F will thus follow the steps described below:

$$F \leftrightarrow F^d \leftrightarrow F^{d+1} \leftrightarrow \dots \leftrightarrow F^{d+e} \rightarrow G^{d+e} \leftrightarrow G^{d+e-1} \leftrightarrow \dots \leftrightarrow G^d. \tag{6.5}$$

7. Logic is neither analytic nor tautological

The theory of distributive normal forms for first-order logic grounds Hintikka's thesis that there is a class of polyadic first-order inferences that are both synthetic and informative. Consider the issue of syntheticity first. The method of proof from one assumption described above provides a technical definition of sense iii, that Hintikka calls the «*explicit form of sense iii*»[64], and a method to discern which inferences are analytic and which are synthetic. In the former case, no increase in depth is needed: the elimination of trivially inconsistent constituents of depth d is sufficient to show that all the constituents of F^d are among those of G^d. In the latter case, in order to bring out the desired relationship between F and G, an increase in depth is required instead: it is necessary to consider configurations of individuals of greater complexity than those that represent the premise and the conclusion of the argument. This classification of logical inferences allows Hintikka to give a characterization of syntheticity as a matter of degree: an inference of G from F is synthetic of degree e if and only if it is necessary to expand the normal forms of F and G by splitting their constituents into disjunctions of depth $d + e$. In other words, the degree of syntheticity of an inference counts the number of individuals that must be included in the initial configurations in order to be able to derive the conclusion from the premise.

Consider now the issue of the informativeness of logic. Hintikka defines two measures of information content depending on how the probability is assigned to first-order constituents:

i. *Depth information* is the measure obtained by assigning a positive probability weight only to the constituents of the polyadic calculus that are consistent and is equivalent to the notion of semantic information chosen by Bar-Hillel and Carnap[65]. Due to the undecidability of first-order logic, depth information is non-recursive in character, which means that it is not calculable in practice and that in general there is no decision procedure through which the initial distribution of probability can be assigned.
ii. *Surface information* is the measure obtained by assigning non-zero weights to all the constituents that are not trivially inconsistent at a certain depth. Being calculable and realistic, it represents Hintikka's

[64] Hintikka (1973, 185).
[65] Bar-Hillel and Carnap (1953).

> reaction against the absurdity and the useless in practice of measures of information that are not recursive[66]. Moreover, logical deduction might increase surface information, because it might reveal that certain non-trivially inconsistent constituents were nevertheless inconsistent at a greater depth. For this reason, surface information vindicates the idea that not all logical inferences are tautological.

Hintikka's impressive challenge against the logical empiricist claim that logic is both analytic and tautological is marred by several problems. First of all, the disproof method via distributive normal forms is rather complex or, to use Veikko Rantala and Vitali Tselishchev words[67], it is «not very practical» in practice. Now, this feature calls Hintikka's success in having explicated the amount of semantic information generated by deductive inferences into question. Even if Hintikka probably understood his theory to be independent of the actual proof procedure used in the derivation[68], still there are some technical difficulties. The calculation of the surface information of an inference depends on the form of the particular formula involved: as a result, one inference could be made more informative than another by simply adding in irrelevant steps. Moreover, surface probability is only assigned to closed constituents: this means that Hintikka does not define the notion of surface information for formulae containing free variables[69].

Second, the thesis that a class of polyadic inferences of first-order logic is synthetic is, for Hintikka, a vindication of Kant's position. For sure it is a vindication of Kant's talking of mathematics. This is because modern first-order logic includes modes of reasoning that Kant wouldn't have called logical, but mathematical, and it is precisely this kind of derivations that Hintikka considers to be synthetic[70]. Nevertheless, Hintikka's vindication of Kant's thought is only partial and confined to the status of mathematics. As far as logic is concerned, it might be shown that Kant does not apply the analytic-synthetic distinction to logic at all and that for him logical judgments are neither analytic nor synthetic[71]. Thus, as for analyticities, Hintikka is not vindicating Kant's position, but rather the thesis that a long

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[66] Hintikka (1973, 228).
[67] Rantala and Tselishchev (1987, 89).
[68] See Sequoiah-Grayson (2008, 87) and Rantala and Tselishchev (1987, 87).
[69] See Rantala and Tselishchev (1987, 87-8).
[70] Hintikka (1973, 182).
[71] See Larese (forthcoming).

historical tradition culminating with the Vienna Circle implicitly ascribed to the author of the *Critique*.

The third worry about Hintikka's theory is perhaps the most serious one. The set of formulae that turn out to be analytic following Hintikka's definitions is wider than what it might first appear[72]. It includes, beyond many polyadic deductions, also the entire set not only of propositional logic, but also of monadic logic. Because both the propositional and the monadic calculus contain only consistent constituents, the inferences included in this set fail to increase surface information. Hintikka's thesis might be charged of the same accuses that the Finnish logician directed against the Vienna Circle: measures of information that are not feasibly calculable are as absurd as measures of information that are not effectively calculable and the probable intractability of propositional logic, like the undecidability of first-order logic, is a good reason for dismissing the principle of analyticity of logic to a wider level[73].

8. Some difficulties of Hintikka's picture

The analysis above has shown some difficulties that affect each of the various components of Hintikka's overall theory. Section 2 has pointed out that iii.a-iii.e seem to be different senses, rather than variations of the same iii (D1). Sections 3 and 4 have claimed that Hintikka's interpretation of Kant's theories of intuition and analyticity is based on a too radical and sometimes inaccurate reading of the original texts (D2). Section 7 has argued that Hintikka's theory of distributive normal forms is characterized by such a deep level of complexity that it cannot be used in practice (D3).

At the same time, the investigation above has observed certain problems that arise when Hintikka connects the different components of his theory into a single whole. First, the identification between Hintikka's sense iii and Kant's conception of analyticity, which is based on the most controversial elements of his interpretation of the Kantian materials, rests on an analogy between the rule of the existential instantiation and the process known as *ecthesis*. But it is not the case that in general an application of this rule through the introduction of a new individual, that for Kant might be seen as the paradigmatic example of synthesis, increases the degree of the conclusion with respect to its premise (D4).

[72] See Sequoiah-Grayson (2008, 88 and ff.) and D'Agostino and Floridi (2009, 278).
[73] See Hintikka (1973, 228) and D'Agostino and Floridi (2009, 279).

The second worry regards Hintikka's thesis that a certain class of polyadic inferences of first-order logic are synthetic or, to be more precise, the flip side of this reasoning, namely the claim that the remaining class of logical inferences are analytic. On the one hand, Hintikka is vindicating Kant's position about the status of mathematics, but not his original standpoint on logic (D5). On the other hand, Hintikka's class of analyticities is too wide: it includes, beyond many polyadic truths, all propositional and monadic inferences (D6).

Difficulties D4, D5 and D6 arise because Hintikka tries to keep together different languages and issues. Hintikka's reconstruction of the Kantian material might be seen as a way to let Kant speak the same modern language of logical empiricism. The aim of this strategy is to pass off the modern tools used to attack the position of the Vienna Circle as deeply Kantian means and to disguise Hintikka's theory as Kant's defense of his own position through his own equipment. The side effects of this plan are that the resulting theory is Kantian not to a literal, but only to a general sense, and that the criticism against the principle of the analyticity of logic is incomplete.

But these conclusions shed new light on sense iii itself, the kernel of Hintikka's theory of analyticity, which is based on the brilliant idea of considering individuals instead of concepts in order to apply the analytic-synthetic distinction to first-order logic. In particular, doubts arise on various characterizing features of sense iii.e, the last and most favourite version of Hintikka's theory.

(D7) As it has been shown in Section 2, in determining whether an inference is analytic or not, sense iii.e, unlike iii.d, takes into account not only its premise, but also its conclusion, disregarding the direction of the argument. According to Hintikka[74], there are two positive consequences of this choice. First, the proofs from a premise P to a conclusion C and its contrapositive, namely from $\neg C$ to $\neg P$, are either both analytic or both synthetic. Second, it is possible to prove analytically the implication $P \rightarrow C$ from a propositional tautology of the same degree if and only if it is possible to prove C from P analytically.

It is not clear however why the first consequence should be especially welcome. Consider the case in which $P = \forall x \exists y Rxy$ and $C = \forall x \exists y \exists z (Rxy \wedge Ryz)$. Here, the degrees of P and C that, intuitively speaking, amount to the numbers of individuals needed to understand the two sentences, are two and three respectively. Now, in cases like this one,

[74] Hintikka (1973, 193-4).

why should a derivation and its contrapositive be either both analytic or both synthetic? Concluding C from P requires the introduction of a new individual; on the contrary, it is sufficient to analyse the relations between the three individuals occurring in $\neg C$ to conclude that $\neg P$.

As far as the second consequence is concerned, it is surely important to establish a connection between the status of the logical validity $P \rightarrow C$ and the proof of C from P. But Hintikka reaches this result using wrong means. In particular, instead of saying that logical truths of quantification theory are analytic whenever they can be proved from the empty set of assumptions without introducing any new individuals, he stipulates that logical truths of quantification theory are analytic if they can be proved without making use of sentences of a degree higher than that of the sentence to be proved[75]. Hintikka's definition, which is quite unnatural, aims at making it possible for logical validities that are not propositional truths to be analytic. But consider a logical validity such as $\forall x Ax \vee \exists x \neg Ax$. This formula, contrary to what Hintikka seems to suggest, must be taken to be synthetic according to sense iii, because it is necessary to introduce an individual, which is of course new with respect to the empty set of assumptions, to become aware of its validity.

The reason why Hintikka prefers sense iii.e to sense iii.d is probably that the former is closer than the latter to the proof procedure based on the distributive normal forms outlined in Section 6. But, in considering the conclusion of the argument, sense iii.e clearly departs from the original idea of sense iii.

(D8) A different matter, this time concerning not the conclusion, but the premise of a proof, is that Hintikka considers only one-premise arguments and it is not clear the way in which he thought to extend his theory. Following Hintikka's insight that every quantifier invites us to consider one different individual[76], it seems that the degrees of different premises should be added together. But consider as an example the set of premises $\{\forall x Px, \forall x Qx\}$. In order to conceive these sentences, it is sufficient to employ one single individual and not two. As a result, sense iii seems to suggest that it is necessary to consider the degree of the conjunction of the premises, instead of the sum of the degrees of the premises.

(D9) Section 5 above has pointed out that Kant would have conceived every single application of both the universal and the existential instantiation as synthetic steps and that Hintikka, on the contrary, does not

[75] Hintikka (1973, 144).
[76] Hintikka (1973, 139).

support the same standpoint with his sense iii.e. However, he does not clarify the precise role of these patterns of reasoning in his theory. In particular, when does an application of, say, the rule of the existential instantiation increase the number of individuals mutually related? Hintikka does not answer the question. He only suggests that, to have a more realistic measure of the number of individuals that are involved in an application of such a rule, one has to add to the degree of the premise also the number of individuals that occur above in the proof. This is because, in choosing a parameter for the conclusion of such a step, one has to consider all the individuals occurring above in order to be sure that the parameter is new[77]. However, this insight is not developed further. Nor does Hintikka delve into the similar question regarding the role of the universal instantiation.

(D10) Last, the notion of the degree of a formula might probably be improved upon[78]. Not only because, as Hintikka himself recognizes[79], it counts as distinct also individuals that are not connected, such as in the formula $\forall x \exists y (Fx \wedge Gy)$. But also because two parallel existential quantifiers invite us to consider two distinct individuals instead of one, such as in the formula $\exists x Fx \wedge \exists x \neg Fx$. Nevertheless, a more serious objection concerns not so much the exact definition of the degree of a formula, but the way in which this concept is used in measuring the number of new individuals introduced in a given argument. It might be the case that in a certain derivation it is necessary to introduce a number of distinct individuals n such that n is higher than the degree of the premise and of the conclusion and, at the same time, no intermediate stage has degree n, because the n individuals are never mentioned together in the same step[80]. Sense iii.e, as well as sense iii.d, would classify arguments like these ones as analytic. But they are clearly synthetic according to sense iii. This criticism suggests an alternative account, according to which an inference is analytic

[77] Hintikka (1973, 183).
[78] See Van Benthem (1974, 421-2).
[79] See Hintikka (1973, 142, note 33).
[80] An example of such an argument is the derivation from premises P_1, P_2 and P_3 to conclusion C:

$$P_1 = \forall x \forall y \big(Rxy \rightarrow \exists z (Gxz \wedge Gzy)\big)$$
$$P_2 = \forall x \forall y \big(Gxy \rightarrow \exists z (Bxz \wedge Bzy)\big)$$
$$P_3 = \forall x \forall y \big((Bxy \wedge Cx) \rightarrow Cy\big)$$
$$C = \forall x \forall y \big((Rxy \wedge Cx) \rightarrow Cy\big).$$

This example has been presented first by Boolos (1984) and then by Hazen (1999, 86 and ff.) and is discussed in D'Agostino, Larese and Modgil (forthcoming).

if the number of individuals introduced into the derivation is not higher than the degree of the premises.

9. Conclusion

Section 7 has shown that Hintikka classifies as analytic the entire set of valid inferences of propositional logic (D6), although the decision problem for Boolean logic is (most probably) intractable, that is to say, undecidable in practice. This observation was at the origin of the approach of Depth-Bounded Boolean Logics[81]. In this approach the standard semantics of the Boolean operators is replaced by a weaker *informational semantics* whereby the meaning of a logical operator $*$ is fixed by specifying the sufficient and the necessary conditions for an agent to actually possess the information that a $*$-sentence is true (respectively false). An inference is analytic if its conclusion can be established in terms of the actual information that is implicitly contained in its premises according to this weaker explanation of the logical operators. Synthetic inferences are those that essentially require the introduction of *virtual information*, i.e., information that we do not actually possess, but must be temporarily assumed in order to reach the conclusion (as, for example, in the discharge rules of natural deduction).

A recent work[82] proposes a unified treatment of classical first-order logic that brings together the main insights of Depth-Bounded Boolean Logics and Hintikka's idea that synthetic arguments introduce new individuals into the discussion. The resulting approach, called Depth-Bounded First-Order Logics, might be seen as the *pars construens* of the present paper, since it extends the informational semantics to the standard quantifiers according to a further development of Hintikka's sense iii that manages to avoid the difficulties connected with sense iii.e. This approach also specifies something that Hintikka did not clarify enough, namely the circumstances in which the use of the rules of universal and existential instantiation does not introduce new individuals into the argument (see D9). Assume that different quantifiers bind distinct variables[83]. Then, an application of the rule of universal instantiation

[81] D'Agostino (2015), D'Agostino, Finger and Gabbay (2013), D'Agostino and Floridi (2009).
[82] D'Agostino, Larese and Modgil (forthcoming).
[83] The two requirements below on the rules of instantiation have been formulated under the assumption that the premises are in a particular prenex normal form. This assumption does

$$\frac{\forall x F}{F(x/a)}$$

(9.1)

is analytic if a is any parameter that already occurs above in the proof; or else a is a new parameter, provided that no other parameter has been already introduced above by an application of the same rule to a formula of the form $\forall x G$. Loosely speaking, each universal quantifier $\forall x$ can "be used" unrestrictedly if it introduces an individual that already occurs in the derivation, but it can "be used" only once if it introduces an individual that is new. Moreover, an application of the rule of existential instantiation

$$\frac{\exists x F}{F(x/a)}$$

(9.2)

is analytic provided that a is a new parameter and no other parameter has been already introduced above by an application of the same rule to a formula of the form $\exists x F$. Roughly put, each existential quantifier $\exists x$ can "be used" only once.

What's more, the development of sense iii provided by Depth-Bounded First-Order Logics does not consider the individuals occurring in the conclusion in establishing whether the argument is analytic or synthetic (see D7) and admits inferences with more than one premise (see D8). Moreover, it abandons the notion of degree of a formula considering instead the number of individuals that are actually used in the derivation (see D10) and avoids the use of complex and unpractical means such as Hintikka's distributive normal forms for first-order logic (see D3).

But the major strength of this approach is that it takes into serious account the probable intractability of propositional logic and classifies as synthetic not only polyadic inferences as Hintikka does, but also propositional and monadic inferences in which the conclusion is reached through an essential use of information that is not actually contained in the premises (see D5 and D6). In this way, the logical positivistic tenet that logic is analytic has been completely defeated.

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not imply any loss of generality, but it simply aims at keeping the technicalities to the minimum. See D'Agostino, Larese and Modgil (forthcoming).

References

Acerbi, F., 2011, «Perché una dimostrazione geometrica greca è generale», in *La scienza antica e la sua tradizione*, Micheli, G., and Repellini, F.F., eds., Cisalpino, Milano, pp. 25-80.

Acerbi, F., 2019, «Mathematical Generality, Letter-Labels, and All that», in *Phronesis*, 65 (1), pp. 1-49.

Bar-Hillel, Y., and Carnap, R., 1952, «Semantic information», in *The British Journal for the Philosophy of Science*, 4 (14), pp. 147-57.

Boolos, G., 1984, «Don't Eliminate Cut», in *Journal of Philosophical Logic*, 13, pp. 373-8.

Brittan, G.G., 2006, «Kant's Philosophy of Mathematics», in *A Companion to Kant*, Graham, B., ed., Oxford, Blackwell, pp. 222-35.

Brittan, G.G., 2015, *Kant's Theory of Science*, Princeton, Princeton University Press.

Carnap, R., 1956, *Meaning and Necessity*, Chicago, University of Chicago Press.

Carnap, R., Hahn, H., and Neurath, O., 1973, «The Scientific Conception of the World: The Vienna Circle», in Neurath, O., *Empiricism and Sociology*, Neurath, M., and Cohen, R.S., eds., Dordrecht, D. Reidel, pp. 299-328.

Church, A., 1936, «An Unsolvable Problem of Elementary Number Theory», in *American Journal of Mathematics*, 58 (2), pp. 345-63.

D'Agostino, M., 2015, «An Informational View of Classical Logic», in *Theoretical Computer Science*, 606, pp. 79-97.

D'Agostino, M., Finger, M., and Gabbay, D., 2013, «Semantics and Proof-Theory of Depth Bounded Boolean Logics», in *Theoretical Computer Science*, 480, pp. 43-68.

D'Agostino, M., and Floridi, L., 2009, «The Enduring Scandal of Deduction. Is Propositional Logic Really Uninformative?», in *Synthese*, 167 (2), pp. 271-315.

D'Agostino, M., Larese, C., and Modgil, S., forthcoming, «Towards Depth-Bounded Natural Deduction for Classical First-Order Logic», to appear in *Journal of Applied Logics*.

De Jong, W.R., 1997, «Kant's Theory of Geometrical Reasoning and the Analytic-Synthetic Distinction. On Hintikka's Interpretation of Kant's Philosophy of Mathematics», in *Studies in History and Philosophy of Science*, 28 (1), pp. 141-66.

Frege, G., 1879, *Begriffsschrift. Eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*, Halle, Louis Nebert Verlag.

Friedman, M., 2000, «Geometry, Construction, and Intuition in Kant and his Successors», in *Between Logic and Intuition: Essays in Honor of Charles Parsons*, Sher, G., and Tieszen, R.L., eds., Cambridge, Cambridge University Press, pp. 186-218.

Friedman, M., 2010, «Synthetic History Reconsidered», in *Discourse on a New Method. Reinvigorating the Marriage of History and Philosophy of Science*, Domski, M., and Dickinson, M., eds., Chicago, Open Court Publishing, pp. 573-813.

Hazen, A.P., 1999, «Logic and Analyticity», in *The Nature of Logic*, Varzi, A.C., ed., Stanford, Center for the Study of Language and Information, pp. 79-110.

Hintikka, K.J.J., 1953, *Distributive Normal Forms in the Calculus of Predicates*, *Acta Philosophica Fennica 6*, Helsinki, Societas Philosophica Fennica.

Hintikka, K.J.J., 1965a, «Are Logical Truths Analytic?», in *Philosophical Review*, 74, pp. 178-203.

Hintikka, K.J.J., 1965b, «Distributive Normal Forms in First-Order Logic», in *Formal Systems and Recursive Functions*, Crossley, J.N., and Dummett, M.A.E., eds., *Proceedings of the Eighth Colloquium, Oxford, July 1963*, Amsterdam, North Holland Publishing Company, pp. 48-91.

Hintikka, K.J.J., 1966, «An Analysis of Analyticity», in *Deskription, Analitizität und Existenz*, Weingartner, P., ed., Pustet, Salzburg, pp. 193-214.

Hintikka, K.J.J., 1967, «Kant on the Mathematical Method», in *The Monist*, 51 (3), pp. 352-75.

Hintikka, K.J.J., 1968, «Are Mathematical Truths Synthetic a Priori?», in *The Journal of Philosophy*, 65 (20), pp. 640-51.

Hintikka, K.J.J., 1969, «On Kant's Notion of Intuition (*Anschauung*)», in *The First Critique. Reflections on Kant's Critique of Pure Reason*, Penelhum, T., and MacIntosh, J.J., eds., Belmont, CA, Wadsworth Publishing, pp. 38-54.

Hintikka, K.J.J., 1970, «Surface Information and Depth Information», in *Information and Inference*, Hintikka, K.J.J., and Suppes, P., eds., Amsterdam, North Holland Publishing Company, pp. 263-97.

Hintikka, K.J.J., 1973, *Logic, Language-Games and Information. Kantian Themes in the Philosophy of Logic*, Oxford, Oxford University Press.

Hintikka, K.J.J., 1974, *Knowledge and the Known: Historical Perspective in Epistemology*, Dordrecht, D. Reidel Publishing Co.

Hintikka, K.J.J., 1982, «Kant's Theory of Mathematics Revisited», in *Essays on Kant's Critique of Pure Reason*, Mohanty, J.N., and Shehan, R.W., eds., Oklahoma, University of Oklahoma, pp. 201-15.

Hintikka, K.J.J., 1984, «Kant's Transcendental Method and His Theory of Mathematics», in *Topoi*, 3 (2), pp. 99-108.

Hintikka, K.J.J., 1987, «Self-profile», in *Jaakko Hintikka: A Profile*, Bogdan, R.J., ed., Dordrecht, D. Reidel Publishing Co., pp. 3-40.

Hintikka, K.J.J, and Remes, U., 1974, *The Method of Analysis. Its Geometrical Origin and Its General Significance*, Dordrecht, D. Reidel Publishing Co.

Hintikka, K.J.J, and Remes, U., 1976, «Ancient Geometrical Analysis and Modern Logic», in *Essays in Memory of Imre Lakatos*, Cohen, R.S., Feyerabend, P.K., and Wartofsky, M.W., eds., Dordrecht, D. Reidel Publishing Co., pp. 253-76.

Kant, I., 1997, *Prolegomena to Any Future Metaphysics that May Be Able to Come forward as a Science*, Hartfield, G., ed. and trans., Cambridge, Cambridge University Press.

Kant, I., 1998, *Critique of Pure Reason*, Guyer, P., and Wood, A., ed. and trans., Cambridge, Cambridge University Press.

Kolak, D., and Symons, J., 2004, «Annotated Bibliography of Jaakko Hintikka», in *Quantifiers, Questions and Quantum Physics. Essay on the Philosophy of Jaakko Hintikka*, Kolak, D., and Symons, J., eds., Dordrecht, Springer, pp. 273-355.

Langford, C.H., 1992, «The Notion of Analysis in Moore's Philosophy», in *The Philosophy of G.E. Moore*, Schilpp, P.A., ed., La Salle, Open Court, pp. 321-42.

Larese, C., forthcoming, «Kant on the Analyticity of Logic».

Netz, R., 1999, «Proclus' Division of the Mathematical Proposition into Parts: How and Why Was it Formulated?», in *The Classical Quarterly*, 49 (1), pp. 282-303.

Parsons, C., 1969, «Kant's Philosophy of Arithmetic», in *Philosophy and Method: Essays in Honor of Ernst Nagel*, Morgenbesser, S., Suppes, P., and White, M., eds., New York, St. Martin's Press, pp. 568-94.

Parsons, C., 1980, «Mathematical Intuition», in *Proceedings of the Aristotelian Society*, 80, pp. 145-68.

Parsons, C., 1983, «Kant's Philosophy of Arithmetic», in *Philosophy, Science and Method. Essays in Honor of Ernest Nagel*, Morgenbesser, S., *et al.*, eds., New York, St. Martin's Press, pp. 568-94.

Rantala, V., 1987, «Constituents», in *Jaakko Hintikka: A Profile*, Bogdan, R.J., ed., Dordrecht, D. Reidel Publishing Co., pp. 43-76.

Rantala, V., and Tselishchev, V., 1987, «Surface Information and Analyticity», in *Jaakko Hintikka: A Profile*, Bogdan, R.J., ed., Dordrecht, D. Reidel Publishing Co., pp. 77-90.

Russell, C., 1990, «Hintikka on Kant and Logic», in *Erkenntnis*, 33, pp. 23-38.

Sequoiah-Grayson, S., 2008, «The Scandal of Deduction. Hintikka on the Information Yield of Deductive Inferences», in *Journal of Philosophical Logic*, 37 (1), pp. 67-94.

Turing, A.M., 1937, «On Computable Numbers, with an Application to the *Entscheidungsproblem*», in *Proceedings of the London Mathematical Society* II, 42 (1), pp. 230-65.

Van Benthem, J.F.A.K., 1974, «Hintikka on Analyticity», in *Journal of Philosophical Logic*, 3 (4), pp. 419-31.