



A comparison between relativism and context-shifting semantics for future contingents

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Abstract

John MacFarlane (2003, 2008, and 2014) elaborates a semantics for future contingents – the so-called relativism – in order to capture the two following intuitions. The first one is the indeterminacy intuition: if on Monday it is neither impossible nor unpreventable that there will be a sea battle on the next day, “There will be a sea battle tomorrow” seems to be neither true nor false from Monday’s ‘perspective’. The second one is the determinacy intuition, which suggests that if the sea battle takes place on Tuesday, “There will be a sea battle tomorrow”, used on Monday, is true from Tuesday’s ‘perspective’. This paper presents an alternative characterization of relativism, namely the context-shifting semantics. The latter framework captures both the determinacy intuition and the indeterminacy intuition, and it is identical to relativism up to some technicalities that will be studied in detail. Moreover, since both relativism and the context-shifting semantics modify the truth-at-a-context approach put forward by Kaplan, it will be shown how the two frameworks modify some of the kaplanian principles.

1. Introduction

The problem of future contingents is an ancient philosophical conundrum, which arises when one attempts to identify the truth-conditions of a statement as

(1) Tomorrow there will be a sea battle,

assuming that the present events, along with those that are past, do not determine if the forecast stated in (1) is going to take place or not¹. Since the facts occurred up to the present fail to settle whether tomorrow there is going to be (or not) a sea battle, how should we evaluate (1)?

One of the most recent attempts to interpret future contingents is the Semantic Relativism (SR) purported by John MacFarlane (2003, 2008, 2014). SR is intended to capture two *prima facie* incompatible intuitions. Suppose that someone utters (1) on Monday, and assume that it is unsettled if on Tuesday there will be a sea battle. Then (1) seems to be neither true nor false from Monday's 'perspective'. This is what MacFarlane (2003, 323) calls the indeterminacy intuition. But if on Tuesday the sea battle takes place, statement (1), uttered on Monday, appears to be true from the 'perspective' of Tuesday. This latter evaluation is labeled as the determinacy intuition².

In order to have a unitary framework that reflects both the intuitions, MacFarlane argues that the truth-value of a future contingent is function of a point of evaluation represented by a model, a context of use and a *context of assessment*³. The resulting semantics, SR, implies that a future contingent comes out neither true nor false when assessed at its context of use, as the indeterminacy intuition suggests. However, SR predicts that a future contingent may be either true or false if assessed at a context that differs from the one of its use. In turn, this feature reflects the determinacy intuition.

This paper presents a comparison between SR and what it will be called the context-shifting semantics (CS). The context-shifting framework was originally developed by Bonomi and Del Prete (2008), Del Prete (2009, 2011) and it may be seen as an alternative characterization of SR. CS captures the two intuitions highlighted above by assigning multiple contexts of use to each singular utterance, and it is identical to relativism up to certain technicalities that will be studied in detail. Moreover, since SR and CS modify the truth-at-a-context account put forward by Kaplan (1989), it will be shown how SR and CS reject and conserve some of the kaplanian principles respectively.

¹ The famous sea battle example was given in Aristotle (1941).

² Cfr. MacFarlane (2003, 325)

³ A point of evaluation for a given wff φ is the sequence of the parameters on which the truth-value of φ depends. For example, in alethic modal logic the truth-value of a wff φ depends both on a model, \mathcal{M} , and a possible world, w . Therefore, in the meta-linguistic formula $\mathcal{M}, w \models \varphi$, the sequence $\langle \mathcal{M}, w \rangle$ is the point of evaluation on which the truth of φ depends. For a similar technical vocabulary, see Belnap, Perloff and Xu (2001).

The next section specifies two formal tools. The first one is the language \mathcal{L}_t , whose formulas represent the logical form of tensed statements. The second one is a tree structure \mathfrak{S} , whose role is to give a description of the temporal topology relative to which the statements of \mathcal{L}_t have to be interpreted⁴.

2. The language and the structure

The propositional language that will be adopted, \mathcal{L}_t , is $\mathcal{L}_p \cup \{F(x)\}$. \mathcal{L}_p is the standard propositional language, equipped with the usual logical constants and a countable number of propositional variables: p_0, p_1, \dots, p_n . $F(x)$ is a sentential operator and it has to be read as “in x time units hence, it will be the case that”.

The structure which reflects the notion of open future is a tree and it is defined as follows.

DEFINITION 1. A tree structure is a couple, $\mathfrak{S} = \langle \mathcal{T}, < \rangle$, such that

- \mathcal{T} is a non-empty set of times: t_1, t_2, \dots, t_n ⁵. It will be assumed that \mathcal{T} 's cardinality is infinite and is no more than continuous⁶.
- $<$ is a before-after relation defined on \mathcal{T} satisfying the following properties.
 - $<$ is transitive,
 - $<$ is irreflexive,
 - $<$ is left-connected: $\forall t, t', t'' ((t < t' \& t'' < t') \Rightarrow (t < t'' \text{ or } t'' < t \text{ or } t'' = t))$.
 - $<$ is historical connected: for every $t, t' \in \mathcal{T}$, there is a time t'' such that $t'' \preceq t$ and $t'' \preceq t'$.

⁴The branching-time structure adopted by RS and CS - see Macfarlane (2003, 2008, 2014), Bonomi and Del Prete (2008), Del Prete (2009) - is the one become standard with the works of Thomason (1970), Belnap and Green (1994), Belnap and Perloff and Xu (2001). For recent criticism regarding the relation between branching time and indeterminism, see Rosenkranz (2013).

⁵Often the members of \mathcal{T} are called moments. See Belnap and Green (1994), Belnap and Perloff and Xu (2001).

⁶For a similar choice, see Ciuni (2009). As a conventional remark, t, t', t'' will be used as time variables, where t_1, t_2, t_3 as names for times. The same holds for histories: h, h', h'' are history variables; h_1 is a name for a history.

A history h is a maximal computation in \mathfrak{S} . Equivalently, Zanardo (1998) and Øhrstrøm (2009) define a history as follows.

DEFINITION 2. Given a tree structure \mathfrak{S} , a history h is a set of times satisfying the following conditions.

- For all $t \neq t'$ in h , $t < t'$ or $t' < t$;
- There is no proper superset X of h such that, for all $t \neq t'$ in X , $t < t'$ or $t' < t$.

$H(\mathfrak{S})$ is the set of histories obtainable from \mathfrak{S} . Intuitively, a history represents a temporally complete course of events that may be realized. The notion of history follows the linear interpretation of time by considering every possible path of a tree in isolation from the branching structure. Now let us introduce a function, dur , along the following lines.

DEFINITION 3. Given a tree structure \mathfrak{S} , dur maps couples of times to the set $Y - \{0\}$, where Y is a set of positive numbers whose infinite cardinality is no more than continuous. In addition, dur fulfills the following constraints.

- For every time t , $dur(t, t) = \uparrow$. This condition avoids $x > 0$ as the value of the distance between an arbitrary time and itself, for any x in $Y - \{0\}$.
- For every two times t and t' , $dur(t, t') = dur(t', t)$. The value of the distance between t and t' is the same as the value of the distance between t' and t .
- For every history h , if $t \in h$, there is just a time $t' \in h$ such that $dur(t, t') = x$ and $t < t'$ ($t' < t$). This condition states that there can be just one time $t' \in h$ that has x as the value of its distance from t at the history h and it is later (earlier) than t .

The tree-like diagram in Fig. 1.1 is a partial representation of a tree structure. Given a time t , there is no backward branching at times preceding t , but there can be forward branching at times which are later than t itself.

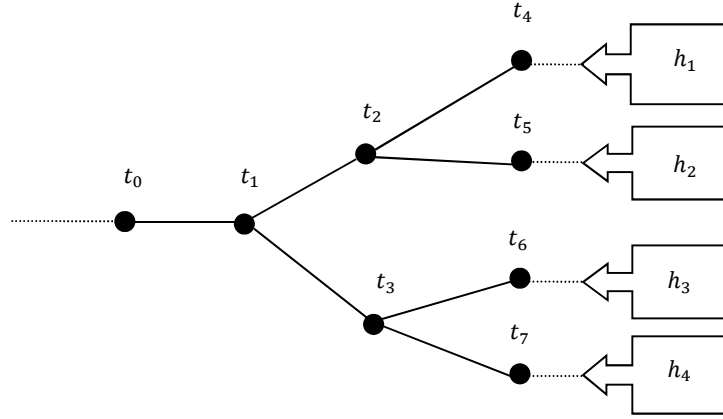


Figure 1.1 A partial representation of a tree structure.

Once that the language and the structure are established, it is possible to clarify MacFarlane's proposal, SR. The following section presents MacFarlane's semantic relativism, highlighting those feature of SR that are relevant for its comparison with CS.

3. Semantic relativism

MacFarlane's proposal divides into two parts: first, SR supplies a proper semantics which consists in a recursive definition of the truth of an arbitrary wff of \mathcal{L}_t at a time and at a history. Then SR assumes a post-semantics (that is, a definition of truth at a context of use and at a context of assessment) in terms of the proper semantics adopted. As it will be clarified later, the definition of the post-semantics in terms of the proper semantics does not amount to identify a context of use with a time and a context of assessment with a history.

In order to explicate SR's two-steps strategy, let us introduce a valuation function V .

DEFINITION 4. A valuation V on \mathfrak{S} maps every propositional variable p_i of \mathcal{L}_t to a subset of $\mathcal{T} \times H(\mathfrak{S})$. V satisfies the following conditions.

- If $\langle t, h \rangle \in V(p_i)$, then $t \in h$.

This condition guarantees that each time-history pair $\langle t, h \rangle$ in $V(p_i)$ is such that h 'passes' through t .

- If $\langle t, h \rangle \in V(p_i)$, then $\langle t, h' \rangle \in V(p_i)$ for any h' such that $t \in h'$.

This condition states that the evaluation of the propositional variables provided by V is history-independent. Indeed if h passes through t and it is such that $\langle t, h \rangle \in V(p_i)$, then $\langle t, h' \rangle \in V(p_i)$ holds for any h' passing through t .

Intuitively, $V(p_i)$ is the set of time-history pairs at which p_i is true under V . Now let us look at an SR-model.

DEFINITION 5. An SR-model is a quadruple, $\mathcal{M}_{SR} = \langle \mathcal{S}, C_{SR}, R, \mathbb{I} \rangle$, where

- \mathcal{S} is a tree structure,
- C_{SR} is a non-empty set of SR-contexts and $C_{SR} \subseteq \mathcal{T}$,
- R is a binary accessibility relation defined on \mathcal{T} ,
- \mathbb{I} is an ockhamist interpretation function⁷.

\mathbb{I} provides SR's proper semantics and it is bivalent, but relative to a time t and a history h .

DEFINITION 6. \mathbb{I} maps triples of wffs of \mathcal{L}_t , times and histories to truth values (1 stands for Ockham-truth and 0 for Ockham-falsity). For every propositional variable p_i , every wffs φ, ψ of \mathcal{L}_t and every time t and history h , such that $t \in h$,

- (i) $\mathbb{I}(p_i, t, h) = 1 \Leftrightarrow \langle t, h \rangle \in V(p_i)$
- (ii) $\mathbb{I}(\sim\varphi, t, h) = 1 \Leftrightarrow \mathbb{I}(\varphi, t, h) = 0$
- (iii) $\mathbb{I}(\varphi \wedge \psi, t, h) = 1 \Leftrightarrow \mathbb{I}(\varphi, t, h) = \mathbb{I}(\psi, t, h) = 1$
- (iv) $\mathbb{I}(\varphi \vee \psi, t, h) = 1 \Leftrightarrow \mathbb{I}(\varphi, t, h) = 1 \text{ or } \mathbb{I}(\psi, t, h) = 1$
- (v) $\mathbb{I}(F(x)(\varphi), t, h) = 1 \Leftrightarrow \exists t' (dur(t, t') = x \ \& \ t < t' \ \& \ t' \in h \ \& \ \mathbb{I}(\varphi, t', h) = 1)$

The conditions (i)-(v) supply the Ockhamist truth-value that a wff of \mathcal{L}_t can have relatively to times and histories. Condition (i) says that p_i is Ockham-true with respect to a time t and a history h just in case $\langle t, h \rangle \in V(p_i)$. But since the evaluation of propositional variables provided by V is history-independent, the evaluation of propositional variables delivered by \mathbb{I} would be history-independent as well. In other terms, $\mathbb{I}(p_i, t, h) = \mathbb{I}(p_i, t, h')$, for any two histories h and h' such that $t \in h$ and $t \in h'$. In particular, the historical parameter occurring in the clauses of Definition 6 plays a crucial role in vir-

⁷ According to Prior (1967), \mathbb{I} represents the formalization of William of Ockham's philosophy of time. For recent criticism on Prior's interpretation, see Øhrstrøm (2009). For a general introduction, see Øhrstrøm and Hasle (1995).

tue of (v). If one uses \mathbb{I} to evaluate a formula of the form $F(x)(\varphi)$ at a time t and at a history h , condition (v) entails that the future operator $F(x)$ shifts the time of evaluation t to a time that is x units later than t . But a tree structure might provide more than one history (among those passing through t) that has a time located at x units in the future of t . Accordingly, the historical parameter in (v) tells us the path along which the shift of the time of evaluation has to occur.

Now MacFarlane wants to define the notion of truth at a context of use and a context of assessment in terms of the ockhamist interpretation function. Since \mathbb{I} evaluates formulas relative to times and histories, one *may be tempted* to identify a context with a couple specifying a time t and a history h (among those passing through t). Furthermore, one *may be tempted* to say that a wff of \mathcal{L}_t is true at a context just in case it is Ockham-true relative to the time of the context and the history of the context. According to MacFarlane this cannot be the case, since this treatment would privilege one history (that is, the history of the context) among those passing through the time of the context. But the very notion of open future implies that all the histories passing through a time t are metaphysically on a par (at t). If there are two histories branching after t , each of them is as possible as the other (at t). If so, none of the histories passing through t can be preferred over the others as the history of the context at t ⁸. These remarks do not arise only because a tree structure allows several histories to pass through the same time. Indeed there are semantic frameworks based on tree structures that identify a context with a time-history couple, as the ‘context-dependent thin red line’ approach does⁹. MacFarlane’s rejection of the idea that a context might determine a unique history is mainly motivated by his philosophical notion of open future. If the future of t is open, there is no fact of the matter that determines at t the history (among those passing through t) that is going to be realized after t . These considerations induce MacFarlane to elaborate the following *Indeterminacy Thesis*:

(IT) In branching time frameworks, a context does not, in general, determine a unique “history of the context”, but at most a class of histories that overlap at the context¹⁰.

⁸ Cfr. MacFarlane (2014, 208).

⁹ Cfr. Belnap, Perloff and Xu (2001, 163).

¹⁰ (IT) is slightly different from the Indeterminacy Thesis proposed in MacFarlane (2014, 208). In particular, (IT) does not mention neither contexts of use, nor branching worlds. The modification is due to the fact that the distinction of the contexts of use from the contexts of assessment does not involve a difference between of two kinds of context. Indeed

Thus, an SR-context *is identified* with a time. Given a structure as \mathfrak{S} , it is easy to check that, for every time t , there is just one set of histories passing through t : $H(t) = \{h: t \in h\}$. In the general case, an SR-context (that is, a time) does not determine a unique history, but a class of histories passing through the context.

Once clarified how SR systematizes the notion of context, its post-semantics runs as follows.

DEFINITION 7. Given a wff φ of \mathcal{L}_t , an SR-model \mathcal{M}_{SR} , an SR-context of use c_U in \mathcal{C}_{SR} and an SR-context of assessment c_A in \mathcal{C}_{SR} ,

$$(7.1) \mathcal{M}_{SR}, c_U, c_A \models_{SR} \varphi \Leftrightarrow R(c_U, c_A) \ \& \ \forall h (h \in H(c_A) \Rightarrow \mathbb{I}(\varphi, c_U, h) = 1)$$

$$(7.2) \mathcal{M}_{SR}, c_U, c_A \not\models_{SR} \varphi \Leftrightarrow R(c_U, c_A) \ \& \ \forall h (h \in H(c_A) \Rightarrow \mathbb{I}(\varphi, c_U, h) = 0)$$

$$(7.3) \text{not}(\mathcal{M}_{SR}, c_U, c_A \models_{SR} \varphi) \ \& \ \text{not}(\mathcal{M}_{SR}, c_U, c_A \not\models_{SR} \varphi) \\ \Updownarrow \\ R(c_U, c_A) \ \& \ \exists h, h' (h, h' \in H(c_A) \ \& \ \mathbb{I}(\varphi, c_U, h) = 1 \ \& \ \mathbb{I}(\varphi, c_U, h') = 0)$$

where

$$(C1) R(c_U, c_A) \Leftrightarrow H(c_A) \subseteq H(c_U)$$

A remarkable feature of (7.1)-(7.3) is that the context of use, c_U , initializes the time at which evaluate a given sentence φ , though the history variables occurring within the truth-conditions of φ range over a domain individuated by the context of assessment, c_A ¹¹. As we shall see, this trait allows SR to predict truth-values transitions of future-tensed statements.

the very same context can be semantically applied both as a context of use and as a context of assessment. See MacFarlane (2008, 90-91). Therefore, the fact that (IT) talks about contexts without qualifying their semantic application is coherent with the roles played by contexts within SR, but, on the other hand, it makes (IT) more general. In (IT) is also omitted the notion of branching worlds, which is substituted with that of history. Clearly this modification can be seen as a terminological variation. Every history represents an entire possible course of events, so a history can be conceived as a possible world maximally extended in time. Accordingly, histories capture the same information conveyed by Macfarlane's branching worlds.

¹¹ Assume that $c_U \succcurlyeq c_A$ and that φ is substituted with a wff of \mathcal{L}_t of the form $F(x)(\phi)$. By (7.1) it follows that $F(x)(\phi)$ (evaluated at c_U, c_A) is equivalent to the formula $AF(x)(\phi)$

Another relevant characteristic of SR is the accessibility relation stated in (C1). Its role is to inform whether a sentence used at a context c_U can be assessed at a context c_A . This relation mimics the epistemic positions that an agent may gain in evaluating future-tensed statements during the flowing of time. In order to appreciate these features, let's look at the following example (Fig. 3.1).

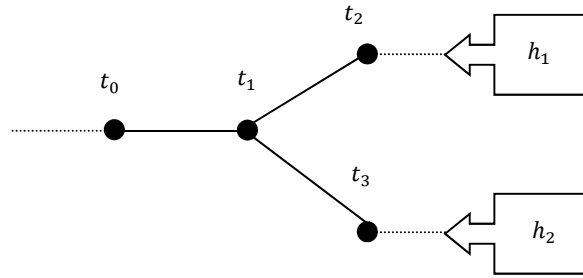


Figure 3.1 A toy example

Suppose that p_1 is Ockham-true at $\langle t_2, h_1 \rangle$ and Ockham-false at $\langle t_3, h_2 \rangle$. Assume that an agent A uses $F(1)(p_1)$ at t_1 . Given (C1), A can assess $F(1)(p_1)$, used at t_1 , from t_1 . Furthermore, suppose that $\text{dur}(t_1, t_2) = \text{dur}(t_1, t_3) = 1$, where both t_2 and t_3 are later than t_1 . Since there are two histories, h_1 and h_2 , in which p_1 has incompatible Ockham-truth-values, (7.3) and (v) imply that $F(1)(p_1)$, used at t_1 and assessed at t_1 , is neither true nor false. This result reflects the fact that the epistemic position that A occupies when she assesses $F(1)(p_1)$ at t_1 is particularly unfavorable, since she cannot know at t_1 what the future may be. In turn, this fact captures the indeterminacy intuition.

But now let us suppose that the world evolves in such a way that h_1 becomes the actual history among those passing through t_1 . Accordingly, since (C1) implies that t_2 is R -accessible from t_1 , (7.1) and (v) guarantee that $F(1)(p_1)$ comes out true if used at t_1 and assessed at t_2 . In turn, this feature captures the determinacy intuition: what is predicted by A using $F(1)(p_1)$ at t_1 is settled at t_2 , so the prediction made at t_1 seems to have a determinate truth-value from the perspective of t_2 .

(evaluated at c_U) by the computational tree logic (CTL) introduced by Clarke and Emerson (1982). In CTL the A operator stands for a universal quantification over the histories passing through the time of evaluation, while $F(x)$ is the metric future operator. It is easy to check that a similar remark applies with respect to clause (9.1) of CS and the semantics provided by CTL.

Let us notice that (C1) entails that, if $t \preceq t'$, then $R(t, t')$. This means that if $t \preceq t'$ holds, t' is a context of assessment for any statement used at t . However, (C1) alone does not exclude that a context of assessment might precede a context of use, for the relation defined in (C1) may be not equivalent with the following one.

$$(C2) R'(t, t') \Leftrightarrow t \preceq t'$$

As it will be shown later, the difference between (C1) and (C2) is quite relevant. Indeed if one substitutes the relation R in (7.1)-(7.3) with the one stated in (C2), one obtains a semantic framework which is identical with CS. This is one of the reasons why CS will be described as an alternative characterization of SR. CS shares with SR several features, including that of reflecting both the indeterminacy intuition and the determinacy intuition. As we shall see in the next two sections, the peculiarity of CS is to assign to a given utterance multiple contexts of use.

4. Times and actuality

The CS framework sticks to two fundamental concepts. The first one concerns the double information conveyed by the members of \mathcal{T} . The second one is about a time-dependent account of actuality.

First of all, each time can encode a two-fold information. Clearly a given time captures a *temporal* information in virtue of the \prec -relations that it bears with itself and with the others members of \mathcal{T} . For instance, the irreflexivity of \prec implies that any time cannot precede or succeed itself. If t and t' are \prec -related and $t \neq t'$, either the former temporally precedes the latter, or the latter temporally precedes the former. Furthermore, if t and t' are not \prec -related and $t \neq t'$, they both succeed an earlier time t'' .

On the other hand, each time can be understood as the temporal location at which several events may take place. Since an event e is a concrete entity, if e occurs at a time t , t is the only time at which e occurs. In this sense, events cannot repeat themselves and they cannot be temporally located at more than one time. If a time keeps track of the events that occur at that time, each node of a tree supplies, along with a temporal information, also a *modal* information. Indeed every t can tell us something about the modal status that several events have, when they are 'seen' from the temporal perspective of t .

For example, given a time t , the set $\{t': t < t'\}$ identifies the temporal locations of those events that are *possible* at t and temporally located after t ¹². But what about actuality?

CS claims that every event occurring at a time belonging to $\{t': t < t'\}$ cannot be actual at t . Given that the set $\{t': t < t'\}$ collects the temporal locations of those events that may happen after t , then these events can become actual after t , but they are not actual before the time at which they occur. One could object that the times at which the actual events are located have to be identified with those collected within a singular history of $H(t)$. However, if there was such a history, it would be metaphysically advantaged relatively to the other members of $H(t)$. But if it were so, it would suggest that there is just one objective possible future stemming from t , and indeterminism would be contradicted.

On the other hand, every time t can be associated with what could be called the Past-Present of t : $\{t': t' < t \text{ or } t' = t\}$. The left-linearity of $<$ implies that the Past-Present of t is linearly ordered by $<$, so that there cannot be branches between its members. Usually this trait corresponds to the uniqueness of the past of t : the absence of branches up to t represents that the past of t is settled. CS reads this peculiarity of a tree-like structure as encoding not only an information concerning the inevitability of the past, but also a feature that tells us something about the notion of actuality. In particular, every event that occurs at a time in the Past-Present of t is not only settled at t , but it is also *actual* at that very same time. The Past-Present of t identifies, along with the temporal locations of the events that are settled at t , also those temporal locations of the events that are actual at t .

In order to explicate this point, let us look at the following example. Queen Elisabeth's birth is a past event relatively to the present. Nonetheless, the event just mentioned is part of our world as it is now, or, in other terms, it counts as actual also at the present time, despite its temporal location falls within the past. In general, an event that is past remains an actual one from the perspective of the present: it remains actual at those times that are later than the time in which it has been occurred.

Moreover, as time goes by, the domain of actual events grows up. But this means that every event that counts as actual at a given time t - every event which occurs at a time in $\{t': t' = t \text{ or } t' < t\}$ - remains actual at every subsequent time of t , whatever the future of t may be. Using the example

¹² Belnap (1996) calls the set $\{t': t < t'\}$ the "future of possibilities" at t . For a similar modal vocabulary, see Zanardo (1998).

sketched above, Bonomi and Del Prete (2008, 13) give the following justification,

[...] consider the event of Queen Elisabeth's birth. That she was born at some place at a given time was true 24 hours ago, is true now and will be true 24 hours from now. [...] the event of her birth is part of the world as it was 24 hours ago, as it is now and as it will be 24 hour from now.

It is possible to clarify this conception of actuality using the figures below.

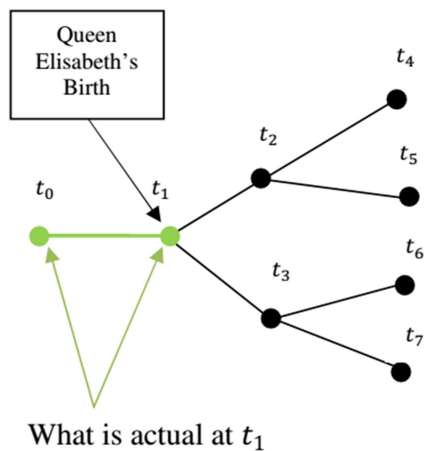


Figure 4.1 Actuality at t_1

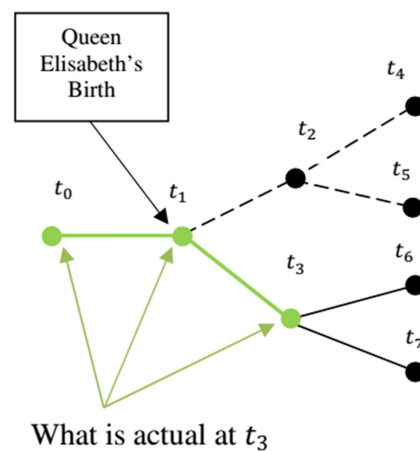


Figure 4.2 Actuality at t_3

Let us suppose that Queen Elisabeth's birth occurs at t_1 . If t_1 is seen as the present (Fig. 4.1), then its future is open, since there are branches stemming from t_1 itself. But every time that occurs after t_1 counts t_1 as an element of its past, so that the event of Queen Elisabeth's birth would be located at every "trunk" of any sub-tree corresponding to each future time accessible from t_1 . Given the account of actuality sketched above, this means that Queen Elisabeth's birth would remain an actual event at every future time accessible from t_1 .

For instance, let us suppose that the temporal development of the world after t_1 makes t_3 the present time (Fig. 4.2). Accordingly, the times collected in the sets $\{t_2, t_4\}$ and $\{t_2, t_5\}$ lose their character of historical possibilities, but the trunk of the sub-tree associated with t_3 (i.e., the Past-Present of t_3) contains t_1 as one of its members. Consequently, since Queen Elisabeth's birth is an event whose temporal location is in the Past-Present of t_3 , then it is an actual event at t_3 as well.

To sum up, every event which occurs at a time in the Past-Present of t is actual at t , and it remains actual at every t' , such that $t < t'$. Nevertheless, if an event e is actual at a time t' , where $t < t'$, it does not follow that e is actual at t . Indeed, e may be temporally located at a time that is later than t . In this case the temporal perspective of t counts e as a possible (but not actual) event.

On that view, the time-dependent notion of actuality has two main characteristics. On the one hand, it is dynamic. As time flows, the domain of actual events grows up, including all those events that occur at times that became past or present. On the other hand, it is 'past-conservative'. Every time conserves the actuality of the events occurred in its past.

CS captures the time-dependent actuality elaborated so far by the following *Principle of Persistence*:

(PP) For every event e and every two times t, t' if e is actual at t and t' is such that $t \preceq t'$, then e is actual at t' as well¹³.

These remarks have a significant impact on CS semantics. First of all, they play an important role in modeling the notion of context of use. Furthermore they allow to predict the truth-value transitions required by the relativist.

5. Context-shifting semantics

In the standard truth-at-a-context framework¹⁴, a context of use for a modal-temporal language specifies (at least) two parameters: the actual world at which the utterance of a given sentence takes places, and the time of that utterance. The conception of a context of use just given is summarized in the Standard Context Principle:

(SP) Let a context of use for an utterance of φ be a couple, $\langle w, t \rangle$, such that w is the world at which the utterance of φ is *actual*, and t is the time of that utterance¹⁵.

Given the modal information conveyed by times, CS reads them as those locations at which several utterance events may be actual. But then the

¹³ Cfr. Bonomi and Del Prete (2008, 13).

¹⁴ Cfr. Kaplan (1989, 544).

¹⁵ Cfr. Bonomi and Del Prete (2008, 14).

value of the parameter of the (actual) world of a context of use can be identified with a time, since every time represents a temporal location at which an utterance of the sentence may count as actual. Accordingly, a CS-context is a couple $c = \langle w_c, t_c \rangle$, where both “ w_c ” and “ t_c ” range over the members of \mathcal{T} , but the value of “ w_c ” is interpreted as the world of c , while the value of “ t_c ” is understood as the time of the context c ¹⁶.

The way in which CS systematizes contexts, along with (PP) and (SP), allows to assign to an utterance of φ multiple contexts of use. Let us assume that an utterance u of φ is actual at a world w_c and temporally located at a time t_c . The correspondent CS-context of use for u would be the pair $\langle w_c, t_c \rangle$. Now the notion of actuality assumed in (PP) implies that every time w' that is either identical with or temporally posterior to w_c counts u as an actual event. This assumption, together with (SP), implies that the utterance u of φ has as its contexts of use those pairs collected in the set $\{\langle w', t_c \rangle : w_c < w' \text{ or } w_c = w'\}$. In general, the account for the notion of a CS-context, in conjunction with (PP) and (SP), implies the following *Conservative Principle*:

(CP) If e is an utterance event and c is a context of use for e , then c' is a context of use for e as well, where c differs from c' at most in its world coordinate, and $w_c \leq w_{c'}$ ¹⁷.

It is worthwhile to notice that the shift of the world-of-use coordinate provided by (CP) does not entail an additional shift involving the time-of-use parameter. The value of the time at which a sentence is used remains fixed once for all. The reason of this restriction is due to the fact that utterances are intended as events that can have only one temporal location. This feature seems to capture something quite intuitive: if one utters “In 24 hours, there will be a sea battle” on Monday at 4 p.m., nothing can change the fact that the sentence is used on Monday at 4 p.m., even if its utterance remains an actual event during the following days.

Given two CS-contexts c and c' , the conservative principle tells us whether an utterance of φ , made at c , remains actual at c' . In other words, (CP) informs us if c' is a context of use for an utterance u of φ , assuming that u is actual at the world of c and located at the time of c . This condition

¹⁶ Since w_c ranges over times, we will represent a CS-context in two ways: $c = \langle w_c, t_c \rangle$ and $c = \langle t, t \rangle$. The important thing is that the first member of a CS-context is intended as its world parameter, where the second member is its time parameter.

¹⁷ Cfr. Bonomi and Del Prete (2008, 14).

can be used to define the class of *proper* contexts of use for an arbitrary utterance of a sentence.

Provided that t is the time at which an utterance event u of φ takes place, for every $t' < t$, u is not actual at t' , but counts as a possible event. In general, this means that u is not actual at those times that precede the one which represents its temporal location. Accordingly, if one sticks to the Standard Context Principle, every time t' , such that $t' < t$, wouldn't be adequate to represent the world-parameter at which u is an actual event. The conclusion is that the *primitive* proper context of use for u is a pair $c = \langle w_c, t_c \rangle$, in which the value of the time variable is the same as the value of the world parameter. Given the Conservative Principle stated above, the set $\{\langle w', t_c \rangle : w_c < w' \text{ or } w_c = w'\}$ is the class of the CS-contexts at which the utterance u of φ remains actual. But then, for every CS-context c , c is a proper context of use for an utterance of φ just in case $t_c = w_c$ or $t_c < w_c$.

A CS-context does not contradict the indeterminacy thesis assumed by SR and stated in (IT). Indeed any world parameter w_c occurring within a CS-context can be univocally associated with a correspondent set, $H(w_c)$. In turn, this set represents all the histories passing through w_c . If there were histories branching at times that are later than w_c , then w_c would belong to every history collected in $H(w_c)$. This means that every CS-context does not single out any privileged history.

Another remarkable feature of CS is that the extension of a sentence φ is function of the proper CS-context of use selected to compute its truth-value. In order to clarify this point, let us look at how CS elaborates its semantics.

DEFINITION 8. A CS-model is a quadruple, $\mathcal{M}_{CS} = \langle \mathfrak{G}, \mathcal{C}_{CS}, R', \mathbb{I} \rangle$, such that

- \mathfrak{G} is a tree structure,
- \mathcal{C}_{CS} is a non-empty set of CS-contexts and $\mathcal{C}_{CS} \subseteq \mathcal{T} \times \mathcal{T}$,
- R' is a binary accessibility relation on \mathcal{T} and it is defined in (C2),
- \mathbb{I} is the ockhamist interpretation function presented in section three.

Since CS applies the same ockhamist interpretation function used by SR, the two frameworks share the same proper semantics. Both SR and CS assume \mathbb{I} to provide a recursive definition of Ockham-truth at a time and at a history.

The post-semantic problem is solved by CS in a similar manner as it was solved within SR: since every CS-context does not single out any privileged history, CS defines the truth at a context quantifying over the set of

histories which overlap at the world of a CS-context. In symbols, the post-semantic account offered by CS is the following.

DEFINITION 9. Given a CS-model, \mathcal{M}_{CS} , a CS-context of use c in C_{CS} and a wff φ of \mathcal{L}_t ,

$$(9.1) \mathcal{M}_{CS}, c \models_{CS} \varphi \Leftrightarrow R'(t_c, w_c) \ \& \ \forall h (h \in H(w_c) \Rightarrow \mathbb{I}(\varphi, t_c, h) = 1)$$

$$(9.2) \mathcal{M}_{CS}, c \not\models_{CS} \varphi \Leftrightarrow R'(t_c, w_c) \ \& \ \forall h (h \in H(w_c) \Rightarrow \mathbb{I}(\varphi, t_c, h) = 0)$$

$$(9.3) \text{not}(\mathcal{M}_{CS}, c \models_{CS} \varphi) \ \& \ \text{not}(\mathcal{M}_{CS}, c \not\models_{CS} \varphi)$$

$$\Downarrow$$

$$R'(t_c, w_c) \ \& \ \exists h, h' (h, h' \in H(w_c) \ \& \ \mathbb{I}(\varphi, t_c, h) = 1 \ \& \ \mathbb{I}(\varphi, t_c, h') = 0)$$

Let us notice that the time of the context t_c in (9.1)-(9.3) plays the same role of that of the context of use within SR's post-semantics. Indeed both the parameters initialize the time at which evaluate a given sentence φ . On the other hand, the world of the context coordinate w_c occurring in (9.1)-(9.3) plays a similar role to that of the context of assessment within SR's post-semantics. In this case, they both identify the domain over which the history variables range in evaluating wffs of \mathcal{L}_t . The accessibility relation R' - occurring in (9.1)-(9.3) and stated in (C2) - guarantees that the CS-context applied to compute the truth-value of φ is a proper one. As we have seen, this means that the CS-context useful to compute the truth-value of φ must count one of its utterances as an actual event, so that the world parameter specified by the context is a time at which that utterance is actual. Given the consideration stated above, the contexts at which a given utterance of φ is actual are those in which the world parameter is either identical or posterior to the time of the utterance. Accordingly, there can be multiple proper contexts of use available to compute the truth-value of an utterance of φ , where this multiplicity is the characteristic that allows CS to predict truth-value transitions of an utterance of a statement.

An application of the CS semantics will clarify this point. Recall the example given before using Figure 3.1, where p_1 is Ockham-true at $\langle t_2, h_1 \rangle$ and Ockham-false at $\langle t_3, h_2 \rangle$. If an agent A utters $F(1)(p_1)$ at $c_1 = \langle t_1, t_1 \rangle$, (C2) tells us that c_1 is a proper context of use for that utterance. If one supposes that $\text{dur}(t_1, t_2) = \text{dur}(t_1, t_3) = 1$, where both t_2 and t_3 are later than t_1 , then (9.3) and (v) imply that $F(1)(p_1)$, used at c_1 , is neither true nor false. This feature reflects the unfavorable epistemic position that A has at c_1 , accounting for the indeterminacy intuition. But now let us assume that

the world evolves in such a way that h_1 becomes the actual history among those which pass through t_1 . (C2) implies that $c_2 = \langle t_2, t_1 \rangle$ is a proper context of use for the utterance of $F(1)(p_1)$ at t_1 , and by (9.1) and (v) it follows that $F(1)(p_1)$ comes out true if used at c_2 . This result captures the idea that A at c_2 has a more favorable epistemic position than the one she had at c_1 . In turn, this characteristic reflects the determinacy intuition.

In the next section it will be studied some of the relations occurring between SR and CS. It will be shown that CS is identical with SR up to some technicalities, and it will be analyzed how the two frameworks modify the Kaplanian truth-at-a-context account.

6. The comparison and some morals

As we have seen, SR and CS are very similar, and they share the following main features:

- both SR and CS assume the same branching-time structure,
- both SR and CS have an Ockhamist proper semantics,
- both SR and CS quantify over histories within their post-semantics,
- both SR and CS captures the indeterminacy intuition and the determinacy intuition.

These similarities suggest that there is a strong relationship between the two frameworks, and the present section is devoted to make that connection explicit. First, we can ask if there is a link between the two accessibility relations used by SR and CS respectively. As a preliminary step, it is useful to look at the following theorem, which follows from the out-set assumptions that characterize a tree-structure \mathfrak{S} and from the definition of R and R' respectively.

(T1) For every tree, $\mathfrak{S} = \langle \mathcal{T}, < \rangle$, if R and R' are defined on \mathcal{T} , then $R' \subseteq R$.

PROOF. Let us assume that there is an arbitrary tree $\mathfrak{S} = \langle \mathcal{T}, < \rangle$ on which both R and R' are defined and $R' \not\subseteq R$. Then there must be two times, $t_1, t_2 \in \mathcal{T}$, such that $t_1 \preceq t_2$, and a history, h_1 , where $t_2 \in h_1$ but $t_1 \notin h_1$. Now $t_1 \preceq t_2$ is equivalent to $t_1 < t_2$ or $t_1 = t_2$. If the latter, by $t_2 \in h_1$ it follows that $t_1 \in h_1$. So one infers the contradiction according to which $t_1 \notin h_1$ and $t_1 \in h_1$. If $t_1 < t_2$, the Pairing Axiom and Union Axiom guarantee that there is a set, $g \subseteq \mathcal{T}$, such that $g = h_1 \cup \{t_1\}$. Therefore, $h_1 \subseteq g$ and

$t_1 \in g$. Since h_1 is linearly ordered by $<$, and given that $h_1 \subseteq g$, $t_2 \in h_1$, $t_1 \in g$ and $t_1 < t_2$, it follows that g is linearly ordered by $<$. But since h_1 is a chain of times which is maximal for inclusion, then $h_1 = g$. Provided that $t_1 \in g$ and $h_1 = g$, it follows that $t_1 \in h_1$. Accordingly, $t_1 \notin h_1$ and $t_1 \in h_1$. By reductio, there is no tree in which both R and R' are defined and $R' \not\subseteq R$. Thus (T1) follows.

However the converse of (T1) does not hold in every structure satisfying the constraints imposed to \mathfrak{S} . This amounts to argue that:

(T2) There is at least a tree $\mathfrak{S} = \langle \mathcal{T}, < \rangle$, such that both R and R' are defined on \mathcal{T} and $R \not\subseteq R'$.

PROOF. A counterexample to the converse of (T1) – or, equivalently, an instance of (T2) – can be observed using Figure 3.1, where it is the case that $H(t_0) \subseteq H(t_1)$ and $R(t_1, t_0)$. Nonetheless, since $t_1 \not\prec t_0$, $R'(t_1, t_0)$ fails to obtain.

What (T1)-(T2) prove is that the two accessibility relations specified respectively by SR and CS are not identical. In particular, it can be shown that the two relations differ only when defined on trees having at least two $<$ -related times without any branching between them. In other terms, if R and R' are both defined on a tree in which there is a ‘deterministic interval’ between different $<$ -related times, then $R \not\subseteq R'$. To see this, let us focus on the following condition.

(C3) $\forall t, t' (t < t' \Rightarrow H(t) \not\subseteq H(t'))$

If a tree satisfies condition (C3), for every two $<$ -related times, the histories passing through an earlier time are more than the histories passing through a later one. This means that every time is a branching point, or that the tree branches at every time. It is possible to prove that

(T3) For every tree $\mathfrak{S} = \langle \mathcal{T}, < \rangle$ satisfying (C3), if both R and R' are defined on \mathcal{T} , then $R \subseteq R'$.

PROOF. Let us suppose that an arbitrary tree $\mathfrak{S} = \langle \mathcal{T}, < \rangle$ satisfies (C3). Furthermore, assume that there are two times in \mathcal{T} , t_1 and t_2 , such that (I) $H(t_2) \subseteq H(t_1)$, (II) $t_1 \not\prec t_2$ and (III) $t_1 \neq t_2$. By (C3), $t_2 < t_1 \Rightarrow H(t_2) \not\subseteq H(t_1)$, and by (I), $t_2 \not\prec t_1$. Accordingly, $t_1 \not\prec t_2$, $t_2 \not\prec t_1$ and

$t_1 \neq t_2$. Given the historical connectedness of $<$, there is a time t_3 such that $t_3 < t_1$ and $t_3 < t_2$. If $t_3 < t_2$, there must be a history, h_1 , where $t_2, t_3 \in h_1$. Since t_1 and t_2 are not $<$ -related and $t_2 \in h_1$, by the definition of history it follows that $t_1 \notin h_1$. Given that $t_2 \in h_1$ and $t_1 \notin h_1$, $H(t_2) \not\subseteq H(t_1)$. The latter claim contradicts (I), so, by reductio, if a tree satisfies (C3), every two times t and t' in that tree fulfill the condition $H(t') \subseteq H(t)$ only if $t \preceq t'$. This is an equivalent claim to that of (T3).

What (T3) shows is that every tree in which each time is a branching point is such that $\subseteq R'$. But since $R' \subseteq R$ holds for every tree on which the two relations are defined, every tree satisfying (C3) makes R and R' identical. Furthermore, if a tree does not fulfill (C3), there must be two times in it, t and t' , such that $t < t'$ and $H(t) \subseteq H(t')$. But then t and t' exemplify a 'deterministic interval' similar to that of Figure 3.1. Therefore, every tree that does not satisfy (C3) is such that R and R' , when defined on it, cannot be identical. This results, along with the following definition, will be useful to compare SR and CS post-semantics.

DEFINITION 10. Given two models, \mathcal{M}_{SR} and \mathcal{M}_{CS} , let us call them *correspondent* just in case they both specify the same tree and the same ockhamist interpretation function.

The symbols \mathcal{M}_{CS}^* and \mathcal{M}_{SR}^* will denote two correspondent models. The definition just specified is helpful to show that:

(T4) For every two correspondent models \mathcal{M}_{SR}^* and \mathcal{M}_{CS}^* , and for every wff φ of \mathcal{L}_t ,

- .1) $\mathcal{M}_{CS}^*, c \models_{CS} \varphi \Rightarrow \mathcal{M}_{SR}^*, t_c, w_c \models_{SR} \varphi$
- .2) $\mathcal{M}_{CS}^*, c \not\models_{CS} \varphi \Rightarrow \mathcal{M}_{SR}^*, t_c, w_c \not\models_{SR} \varphi$
- .3) $\text{not}(\mathcal{M}_{CS}^*, c \models_{CS} \varphi) \ \& \ \text{not}(\mathcal{M}_{CS}^*, c \not\models_{CS} \varphi)$

↓

$$\text{not}(\mathcal{M}_{SR}^*, t_c, w_c \models_{SR} \varphi) \ \& \ \text{not}(\mathcal{M}_{SR}^*, t_c, w_c \not\models_{SR} \varphi)$$

PROOF FOR (T4.1). Let us assume that $\mathcal{M}_{CS}^*, c \models_{CS} \varphi$; by (9.1), $R'(w_c, t_c)$ and $\forall h(h \in H(w_c) \Rightarrow \mathbb{I}(\varphi, t_c, h) = 1)$. By $R'(t_c, w_c)$ and (T1), it follows that $R(t_c, w_c)$. Since $R(t_c, w_c)$ and $\forall h(h \in H(w_c) \Rightarrow \mathbb{I}(\varphi, t_c, h) = 1)$, (7.1) and (9.1) imply that $\mathcal{M}_{SR}^*, t_c, w_c \models_{SR} \varphi$. A similar proof can be given for (T4.2) and (T4.3).

The converses of (T4.1)-(T4.3) hold only with respect to those pairs of correspondent models \mathcal{M}_{SR}^* and \mathcal{M}_{CS}^* specifying a tree which fulfills (C3). Indeed, given (T3), it is easy to check that in every tree fulfilling (C3) the two semantics are perfectly identical.

However, the fact that the converses of (T4.1)-(T4.3) do not hold with respect to every class of correspondent SR and CS-models do not prevent both semantics to identify the same set of valid formulas of \mathcal{L}_t . To see that, let us define the notion of validity relative to SR and CS respectively.

DEFINITION 11. A wff φ of \mathcal{L}_t is SR-valid (in symbols, $\models_{SR} \varphi$) iff, for every SR-model \mathcal{M}_{SR} , every context of use c_U and every context of assessment c_A , $\mathcal{M}_{SR}, c_U, c_A \models_{SR} \varphi$.

DEFINITION 12. A wff φ of \mathcal{L}_t is CS-valid (in symbols, $\models_{CS} \varphi$) iff, for every CS-model \mathcal{M}_{CS} and every CS-context c , $\mathcal{M}_{CS}, c \models_{CS} \varphi$.

Given the definitions just presented, it follows that:

(T5) For every wff φ of \mathcal{L}_t , $\models_{CS} \varphi \Leftrightarrow \models_{SR} \varphi$

PROOF. (\Rightarrow) Suppose that $\models_{CS} \varphi$ but not $\models_{SR} \varphi$. Then there is a point, $\mathcal{M}_{SR}, c_U, c_A$, at which φ is not true. Accordingly, φ is either (I) false or (II) neither true nor false at $\mathcal{M}_{SR}, c_U, c_A$. If case (I) obtains, by (T4.2) and modus tollens, φ is false at \mathcal{M}_{CS}^*, c . If case (II) obtains, by (T4.3) and modus tollens, φ is neither true nor false at \mathcal{M}_{CS}^*, c . Both (I) and (II) imply a contradiction with respect to the assumption that φ is CS-valid. By reductio, if $\models_{CS} \varphi$, then $\models_{SR} \varphi$. (\Leftarrow) Suppose that $\models_{SR} \varphi$ but not $\models_{CS} \varphi$. This implies that there is a point, \mathcal{M}_{CS}, c , at which either (I) φ is false at \mathcal{M}_{CS}, c , or (II) φ is neither true nor false at \mathcal{M}_{CS}, c . If case (I) obtains, let us generate the correspondent SR-model of \mathcal{M}_{CS} , \mathcal{M}_{SR}^* . Since $\mathcal{M}_{CS}, c \models_{CS} \varphi$, (T4.2) implies that $\mathcal{M}_{SR}^*, t_c, w_c \models_{SR} \varphi$. This conclusion contradicts the assumption that $\models_{SR} \varphi$. Since it is easy to check the contradiction which follows from (II), if $\models_{SR} \varphi$, then $\models_{CS} \varphi$.

Now that the main formal relations between SR and CS are clarified, let us look at some remarks. First, there are two ways to make SR identical with CS. One way is to substitute the R occurring in (7.1)-(7.3) with R' . The other way is to consider only those couples of corresponding SR and CS-models based on trees satisfying (C3). Moreover, the differences between (C1) and (C2) suggest the following observation.

At a first glance, if the role played by the accessibility relations is that of representing how an agent may evaluate a future-tense statement during the flowing of time, it could be argued that R satisfies this task better than R' . Indeed, given that the tree has at least a deterministic interval between two of its times, R allows to assess a statement even before the time of its use. This fact, which is forbidden by R' , seems to be reflected by many linguistic practices, as exemplified by the following claim.

(2) Tomorrow you'll say that there will be a sea battle, and what you'll say would be untrue (if assessed now).

It should be noted that SR allows to assess a statement at a time that is earlier than the one of its use only if the temporal structure does not satisfy (C3). As stated in (T1) and (T3), if a tree branches at every time, the accessibility relation in SR becomes identical with that of CS. Consequently, for every tree fulfilling (C3), the definition of R entails that a statement φ , used at a given time, can be assessed at times that are identical with or later than the time of the utterance of φ . Now SR is a semantics which aims to be as neutral as possible with respect to any metaphysical issue, providing the truth-conditions of future contingents in a way that is coherent with our ordinary talk¹⁸. It is plausible to think that the considerations regarding ordinary linguistic practices do not permit, at least directly, to conclude any metaphysical thesis about the objective structure of reality. In particular, linguistic considerations do not seem to represent a firm base to conclude that the structure of time must exemplify a given feature. Therefore, if the data related to the use of ordinary language are the only ones that guide the development of SR, these data cannot be used to determine whether the structure of time meets or falsifies (C3). This means that an advocate of SR cannot defend the assumption of R arguing that competent speakers are able to compute the truth-value of a sentence from a context of assessment that is earlier than the time of its use. This, indeed, would commit a relativist to argue in favor of (C3) – i.e., in favor of a specific feature that the structure of time should exemplify – on the basis of linguistic data. Accordingly, the accessibility relation provided by SR seems as good as (or as wrong as) that of CS.

Another remark is that both SR and CS modify Kaplan's truth-at-a-context account. The semantic approach put forward by Kaplan (1989) fol-

¹⁸ Cfr. MacFarlane (2014, 54-55)

lows – among others – two general norms. These two rules can be summarized as follows.

(K1) If φ is used at a context c , then c supplies the initial values of every parameter occurring in the truth-conditions of φ .

(K2) Each utterance u of a statement φ has only one context of use.

SR rejects (K1), while CS conserves the very same rule. If one looks at (7.1)-(7.3), one will note that a context of assessment identifies the histories at which evaluate a given statement. In the general case, these histories cannot be individuated using only the context of use. Obviously SR allows to identify a context of use with a context of assessment, and, in this particular case, all the post-semantic work is done by the context of use. However, the relevant semantic phenomenon captured by SR (that is, the truth-value transitions of a future contingent) is met by restricting the initialization role ascribed to the context of use relative to the truth-conditions of a sentence used at that context. This restriction leaves room for other contexts to determine the remaining values occurring in the very same truth-conditions, and this function is covered by the context of assessment.

On the other hand, CS does not contradict (K1), since every proper CS-context of use supplies all the values required to compute the truth-value of a sentence used at that context. Nevertheless CS rejects (K2), where SR conserves it. CS allows truth-value transitions by assigning multiple contexts of use to a singular utterance of a sentence, where SR does not predict any shift of the context of use.

These relations suggest that, if one wants to account for truth-value transitions of future contingents modifying the kaplanian approach, one needs to give up either (K1) – restricting the ‘initializer’ job of a context of use – or (K2) – assigning multiple contexts of use an utterance of a sentence.

In conclusion, it has been shown the main formal relations that SR bears with CS. This latter framework is an alternative characterization of the relativist semantics and it is apt to capture both the determinacy intuition and the determinacy intuition. Moreover, if one wants to allow truth-value transitions of future contingents by modifying the kaplanian semantics, one has to reject either (K1) or (K2).

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